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A hyperchaotic map with grid sinusoidal cavity

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ABSTRACT

Based on closed-loop modulation coupling pattern and the model of sinusoidal cavity, a high-dimensional sinusoidal cavity hyperchaotic system is proposed. The number of sinusoidal cavities is controlled by the system parameters. By designing a piecewise-linear controller, the grid sinusoidal cavity attractors are obtained. The equilibrium points are theoretically analyzed through mathematical calculation. Taking the two-dimensional grid sinusoidal cavity hyperchaotic map as an example, dynamics of the system are analyzed by phase diagram, equilibrium points, Lyapunov exponents spectrum, bifurcation diagram, complexity and distribution characteristics. The results show that it has rich dynamical behaviors, including complicated phase space trajectory, hyperchaotic behavior, large maximum Lyapunov exponent and typical bifurcations. The proposed hyperchaotic map has advantages in complexity and distribution in the whole parameter space. Therefore, it has good application prospects in secure communication.

1. Introduction

Existing chaotic maps can be classified into two categories: one-dimensional (1D) chaotic map and high- dimensional (HD) chaotic map. With the development of computer and cloud computing technology, 1D chaotic map based encryption system cannot ensure the information security [1–3] due to the small key space, the simple orbits and structures. On the contrary, HD hyperchaotic map has excellent properties of unpredictability, ergodicity and sensitivity to their parameters and initial conditions [4], such as Arnolds [5], Hénon [6] and Folded-Towel [7]. They have more complex structures with different variables, parameters, and better chaotic performance. It is much difficult to predict the chaotic orbits by chaotic signal estimation technologies [8,9]. Thus it attracts much attentions and has been widely used in secure communications, image encryption and pseudo-random sequence generator applications [10,11]. Therefore, it is interesting to design a HD chaotic map with better performance.

In recent years, some new or enhanced chaotic maps have been proposed by dimension expansion [12–15], cascade chaos [16,17], physics modeling [18] and so on [19–21]. Some chaotic maps are derived from 1D chaotic map. For example, Li and Liu [12] extended the 1D-Chebyshev into 2D-Chebyshev, but its complexity is not improved. Wu [13] extended the classical 1D-Logistic map into 2D-Logistic map by establishing a close-loop coupling mechanism. Its complexity is a little bit higher, but the structure is rather simple. Based on this method, Hua et al. [14] proposed a 2D Sine Logistic modulation map (2D-SLMM) and a 2D Logistic-adjusted-Sine map (2D-LASM), which introduced a cascade chaotic system by connecting two 1D chaotic maps in series. Although it has hyperchaotic behavior, its MLE is relatively small. Wang and Yuan [16] proposed Logistic-Logistic (LL), Logistic-Cubic (LC) and Logistic-Tent (LT) maps, and it shows that the cascade chaos can increase the LEs values. In addition, based on a special physical mode, Sheng et al. [18] proposed a tangent-delay ellipse reflecting caving map system (TD-ERCS). It has high complexity and zero correlation in total parameter range, but its implementation cost is relatively high. Zhang and Wang [19] proposed a non-adjacent coupled map lattices (NCML). It shows less periodic windows and larger range of parameters, but the complexity is not very high. In this paper, we try to design a HD chaotic map to meet the following requirements: high dimension, excellent chaotic performances, and low implementation cost. On the other hand, to obtain strengthened continuous chaotic system, researchers have put forward the piecewise-linear method, staircase method and sign function method to produce multi-scroll attractors [22–26]. Those chaotic attractors exhibit complicated structures, rich dynamics, multi-scrolls and good applications. An interesting question is whether these design methods can be generalized to the design of discrete high-dimensional chaotic map.

In this paper, we focus on designing a grid sinusoidal cavity hyperchaotic map and analyze its performance. The rest of the pa-

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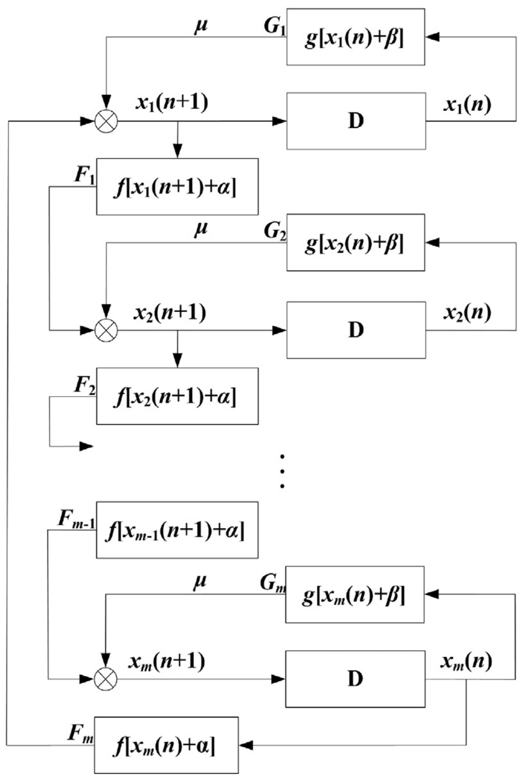

Fig. 1. Modulation-coupling model.

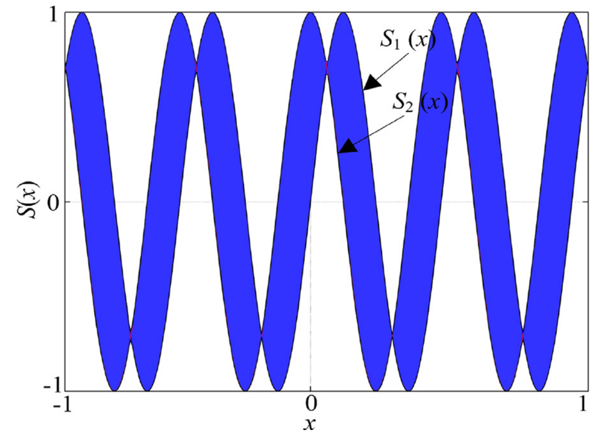

Fig. 2. Diagram of sinusoidal cavity.

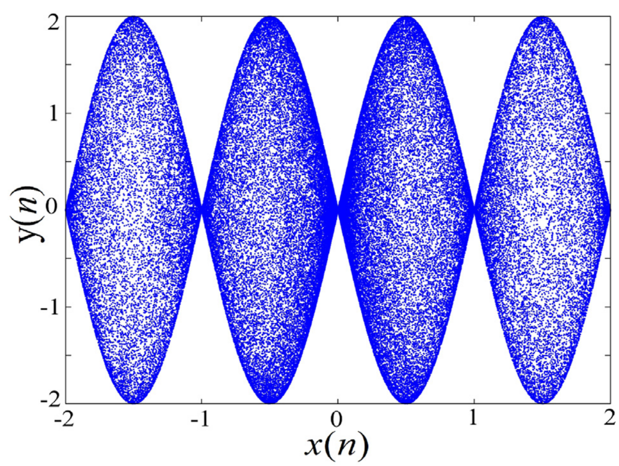

Fig. 3. Phase diagram with $a = 2$, $b = 50$.

per is organized as follows. In Section 2, the constructing method of the grid sinusoidal cavity hyperchaotic map is presented and the equilibrium points are analyzed. In Section 3, we analyze the dynamics and performance of the two-dimensional grid sinusoidal cavity hyperchaotic map. Finally, we summarize the results and indicate future directions.

2. Design of the grid sinusoidal cavity hyperchaotic map

2.1. Close-loop modulation coupling (CMC) model

Consider two different one-dimensional chaotic maps: $g(x)$ and $f(x)$. To improve the randomness and nonlinearity of the system, $f(x)$ is employed to modulate the output of $g(x)$. To make their orbits difficult to be predicted, the dimension is extended to m-dimension, and then a new high-dimensional chaotic map is designed by a close-loop coupling method. The CMC model is obtained by

$$\begin{cases} x_1(n+1) = \mu(f(x_m(n)) + \alpha)(g(x_1(n)) + \beta) \\ x_2(n+1) = \mu(f(x_1(n+1)) + \alpha)(g(x_2(n)) + \beta) \\ \qquad \vdots \\ x_m(n+1) = \mu(f(x_{m-1}(n+1)) + \alpha)(g(x_m(n)) + \beta) \end{cases}, \tag{1}$$

where $x_1, x_2, x_3, \ldots, x_m$ are the state variables, and μ, α and β are the coupling parameters, and m is the lattices ($m \geq 2$). The block diagram of CMC model of system (1) is presented as shown in Fig. 1. Here D is a unit delay. G_{i+1} is used to modulate the output of F_i by a simple multiplication operation, $i = 1, 2, \ldots, m-1$, and G_1 is used to modulate the output of F_m, so all equations are coupled in a closed-loop.

The system has less additions and multiplications in each equation, so the iteration speed of the CMC model is faster than the coupled map lattice (CML) [27] and mixed linear-nonlinear coupled map lattice [28]. Without loss of generality, any two chaotic maps can apply to CMC model, and one is employed to modulate the other, which makes their phase space trajectory more complex.

2.2. Definition of sinusoidal cavity

Suppose there are two sine functions S_1 and S_2, and the curves of the functions intersect a region. The region is named as sinusoidal cavity, and it is denoted as

$$\begin{cases} S_1: \quad s_1 = \tau \sin(\upsilon x + \psi_1) \\ S_2: \quad s_2 = \tau \sin(\upsilon x + \psi_2) \end{cases}, \tag{2}$$

where τ is the amplitude, and υ is the angular frequency, and ψ is the phase. The cavity is shown in Fig. 2.

Derived from Sine map and iterative chaotic map with infinite collapse (ICMIC), Liu et al. proposed a 2D Sine ICMIC modulation map (2D-SIMM) [29,30] and the model is defined by

$$\begin{cases} x_1(n+1) = a\sin[\pi x_2(n)]\sin[b/x_1(n)] \\ x_2(n+1) = a\sin[\pi x_1(n+1)]\sin[b/x_2(n)] \end{cases}, \tag{3}$$

where a, b are system parameters, and a, $b \in (0, +\infty)$. The phase diagram of the system is illustrated as shown in Fig. 3 with $a = 2$ and $b = 50$.

2.2.1. Model of the hyperchaotic map with one row sinusoidal cavity

Based on this method, let $(m, \mu, \alpha, \beta) = (m, a, 0, 0)$, and F_i is chosen to be the Sine map function, which is defined as $f[x_i(n) + \alpha] = \sin[\omega x_i(n)]$. To ensure the solution is uniformly bounded, the equation of G_i is chosen to be the iterative chaotic