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Rogue waves for a generalized nonlinear Schrödinger equation with distributed coefficients in a monomode optical fiber

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A B S T R A C T

Investigated in this paper is the generalized nonlinear Schrödinger equation with distributed coefficients, which describes the amplification or absorption of pulses propagating in a monomode optical fiber with distributed group-velocity dispersion and self-focusing Kerr nonlinearity. By virtue of the Kadomtsev-Petviashvili hierarchy reduction, we obtain the rogue waves based on rogue-wave solutions in terms of the Gramian under certain constraint. We study the effects of group-velocity dispersion, nonlinearity and amplification/absorption coefficients on the rogue waves with the help of figures. Amplitudes of the rogue waves are independent with the group-velocity dispersion and nonlinearity coefficients. The first-order rogue wave with an eye-shaped distribution density and the second-order rogue waves with the highest-peak amplitude and with the triple-peak structure are presented. Both the intermingled or separated composite rogue waves are derived. Periodic rogue waves are obtained and period of the periodic rogue wave increases with the period of the group-velocity dispersion. Furthermore, nonlinear tunneling of the rogue waves is observed: rogue waves get amplified when they reach to the dispersion barriers and recover their original shapes after passing through the barriers, while amplitudes of the rogue waves decrease inside the dispersion wells. Amplification/absorption coefficient influence the background and amplitude of the rogue wave, and three types of the backgrounds are discussed due to different amplification/absorption coefficients.

1. Introduction

Rogue waves, known as the oceanic phenomena responsible for the maritime disasters, have been thought to appear from nowhere and disappear without a trace [1–3]. It has been proved that rogue wave is localized in both space and time with crest heights two times larger than the significant height of its surroundings [4,5]. Experimental realizations of the rogue waves have been achieved in such nonlinear physical systems as those in the atmospheric dynamics [6], plasmas physics [7], Bose–Einstein condensation [8] and nonlinear optics [9,10]. Rational solutions, as the helpful understanding of the rogue-wave phenomena, are a kind of solutions spatially and temporally localized from the background states [11,12]. Dynamics of the rogue waves has been modeled by a nonlinear Schrödinger equation (NLSE) [9,13,14]. The NLSE has not only been used to give a description of slow evolution of a weakly nonlinear wave packet in the deep water [13], but also to describe the light-pulse propagation in the nonlinear optical fiber [9,14]. Rogue waves for the NLSE have been investigated: The first-order rogue waves, also called the Peregrine solitons, have been described [15,18]; People have experimentally studied the characteristics of optical rogue waves [16,17] and devoted themselves to the construction of the rogue-wave solutions of the NLSEs with variable coefficients in the inhomogeneous nonlinear optical fibers [19–26]. Relevant NLSE issues can be seen in Refs. [27–30]. Relevant soliton issues can be seen in Refs. [31–34].

In this paper, we will consider the following generalized NLSE with distributed coefficients in a monomode optical fiber [19,35–41]

$$iu_z - \frac{1}{2}\beta(z)u_{tt} + \gamma(z)u|u|^2 + id(z)u = 0, \tag{1}$$

where $u(z, t)$ is the complex envelope of the electrical field in a comoving frame, z is the normalized propagation distance and t is the retarded time, $\beta(z)$ represents the group-velocity dispersion (GVD) coefficient, $\gamma(z)$ is the nonlinearity coefficient and $d(z)$ is amplification/absorption coefficient. When $\beta(z) = 2$, $\gamma(z) = -2$ and $d(z) = 0$, Eq. (1) can be reduced to the NLSE [9,13–18]. Eq. (1) has described the amplification or absorption of pulses propagating in a monomode optical fiber with distributed GVD

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and self-focusing Kerr nonlinearity [35]. In practical applications, aside from the studies of amplification/absorption and compression/broadening of optical solitons in inhomogeneous systems, Eq. (1) has been used to study the dispersion managed transmission systems [35]. Integrability constraint for Eq. (1) has been derived through the Painlevé test [36]. Multi-soliton solutions [35–40] and self-similar solutions [41] for Eq. (1) have been obtained. Based on the similarity transformation, first-order rogue wave and second-order rogue wave with the highest peak amplitude for Eq. (1) have been derived [19]. In addition, in optical communication and plasmas physics, two special cases of Eq. (1) can be seen as follows:

(1) For the pulse compression technique capable of producing the high-quality 1.3-ps pulses at a repetition rate of 10 GHz, Eq. (1) admits the following form with a reference frame moving at the group velocity [42]:

$$\Psi_\chi = \frac{\alpha(\chi)}{2}\Psi + \frac{i\beta_2}{2}\Psi_{\varsigma\varsigma} - i\gamma\Psi|\Psi|^2 = 0,$$

where Ψ is the field envelope, ς and χ represent the retarded time and propagation distance, $\alpha(\chi)$ is the χ-dependent gain coefficient, β_2 is the GVD and γ is the nonlinear coefficient;

(2) For an unmagnetized dusty plasma consisting of negatively charged dust fluid and ions of two different temperatures, Eq. (1) can be reduced to the cylindrical and spherical geometry-modified NLSE [43]:

$$i\Phi_T + \Phi_{ZZ} + 2\Phi|\Phi|^2 + i\frac{M}{2T}\Phi = 0,$$

where T and Z are the stretched time and radial coordinate, Φ represents the electrostatic wave potential, M represents the cylindrical ($M = 1$) and spherical ($M = 2$) effects.

In this paper, our goal will be focused on the rogue-wave solutions for Eq. (1) through the Kadomtsev-Petviashvili (KP) hierarchy reduction [18,44–46,48], which can construct the higher-order and multi-rogue waves. For example, the KP hierarchy reduction has been used to derive the rogue waves in such constant-variable equations as the NLSE [18], Davey-Stewartson equation [44], KP equation with self-consistent sources [45] and Mel'nikov equation [49].

In Section 2, by virtue of the KP hierarchy reduction, we will obtain the Nth-order rogue-wave solutions in terms of the Gramian for Eq. (1), which have not been seen before, to our knowledge. Based on such solutions, besides the first-order rogue wave and second-order rogue wave with the highest peak amplitude, we also derive the second-order rogue wave with the triple-peak structure, and properties of the rogue waves will be studied. Discussions on the first- and second-order rogue waves will be examined for some choices of the GVD and amplification/absorption coefficients in Section 3. Section 4 will be our conclusions.

2. Bilinear forms and rogue waves for Eq. (1)

2.1. Bilinear forms

Considering the rogue waves that approach the non-zero background at the large z and t, we take the variable transformation

$$u = e^{i\int\gamma(z)e^{-\int 2d(z)\,dz}\,dz}\tilde{u}, \tag{2}$$

where $\tilde{u}(z,t)$ is a complex function of z and t, and rewrite Eq. (1) as

$$i\tilde{u}_z - \frac{1}{2}\beta(z)\tilde{u}_{tt} + \gamma(z)\tilde{u}\left[|\tilde{u}|^2 - e^{-\int 2d(z)\,dz}\right] + id(z)\tilde{u} = 0. \tag{3}$$

Through the dependent variable transformation

$$\tilde{u} = e^{-\int d(z)\,dz}\frac{g(z,t)}{f(z,t)}, \tag{4}$$

where $g(z, t)$ is a complex function and $f(z, t)$ is a real one, Eq. (3) can be transformed into the bilinear forms:

$$\beta(z)D_t^2 g\cdot f - 2iD_z g\cdot f = 0, \tag{5a}$$

$$D_t^2 f\cdot f + 2Kf\cdot f = 2K|g|^2, \tag{5b}$$

under the integrable constraint for Eq. (1) to pass the Painlevé test [36],

$$\gamma(z) = -K\beta(z)e^{2\int d(z)\,dz}, \tag{6}$$

where K is a positive constant, D_z and D_t are the Hirota bilinear differential operators defined by [47]

$$D_z^{\chi_1}D_t^{\chi_2}(F\cdot G) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^{\chi_1}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^{\chi_2} \times F(z,t)\cdot G(z',t')|_{z'=z,t'=t},$$

with χ_1 and χ_2 being the non-negative integers, F as a function of z and t, G as a function of the formal variables z' and t'.

2.2. Rogue waves for Eq. (1)

In order to obtain the rogue waves for Eq. (1), we start with the Gramian expression of the KP hierarchy [18,48] and present the rogue-wave solutions for Eq. (1) in the following Theorem 1.

Theorem 1. *Eq. (1) has the rogue-wave solutions that are given by Transformations (2), (4) and Integrable Constraint (6), where g and f given by the $N\times N$ determinants*

$$u = e^{-\int iK\beta(z)+d(z)\,dz}\frac{g}{f}, \quad g = \tilde{\tau}_1, \quad g^* = \tilde{\tau}_{-1}, \quad f = \tilde{\tau}_0, \tag{7}$$

where

$$\tilde{\tau}_n = \left|m^{(n)}_{2i-1,2j-1}\right|_{1\le i,j\le N}, \ (n = -1, 0, 1)$$

the matrix elements of $\tilde{\tau}_n$ are defined by

$$m^{(n)}_{i,j} = \sum_{k=0}^{i}\sum_{l=0}^{j}\frac{a_k}{(i-k)!}\frac{a_l^*}{(j-l)!}(p\partial_p + \xi' + n)^{i-k} \times (q\partial_q + \eta' - n)^{j-l}\frac{1}{p+q}\bigg|_{p=1,q=1},$$

$$\xi' = p\sqrt{K}t - ip^2K\int\beta(z)\,dz, \quad \eta' = q\sqrt{K}t + iq^2K\int\beta(z)\,dz,$$

j and N are the integers, i in subscript denotes an integer, otherwise $i^2 = -1$, p and q are the complex variables, a_k's are the complex constants, k and l are the positive integers, "" represents the complex conjugation. The proof of Theorem 1 is given in the Appendix A.*

2.2.1. The first-order rogue waves

From Solutions (7), we set $a_0 = 1$ without loss of generality [18], and obtain the first-order rogue waves by taking $N = 1$,

$$u = e^{-\int iK\beta(z)+d(z)\,dz}\frac{m^{(1)}_{11}}{m^{(0)}_{11}}, \tag{8}$$

where

$$m^{(1)}_{11} = \left(\sqrt{K}t - iK\int\beta(z)\,dz + \frac{1}{2} + a_1\right)w \times\left(\sqrt{K}t + iK\int\beta(z)\,dz - \frac{3}{2} + a_1^*\right) + \frac{1}{4},$$

$$m^{(0)}_{11} = \left(\sqrt{K}t - iK\int\beta(z)\,dz - \frac{1}{2} + a_1\right) \times\left(\sqrt{K}t + iK\int\beta(z)\,dz - \frac{1}{2} + a_1^*\right) + \frac{1}{4}.$$