Chaos, Solitons and Fractals 106 (2018) 154–160

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Fuzzy synchronization of chaotic systems via intermittent control

Yuangan Wang[a,b,*], Honglin Yu[a]

[a] Key Laboratory of Optoelectronic Technology and Systems of Education Ministry of China, Chongqing University, Chongqing 400030, PR China
[b] Schoolof Science, Qinzhou University,Qinzhou,Guangxi 535000, PR China

ARTICLE INFO

Article history:
Received 28 June 2017
Revised 10 November 2017
Accepted 15 November 2017

Keyword:
Fuzzy
Synchronization
Chaotic system
Intermittent

ABSTRACT

In this paper, a fuzzy method is combined with intermittent control method to realize synchronization of chaotic system. Two plant rules of intermittent control are considered to get two theorems. Fuzzy scheme for synchronization is proposed in theorem. Finally, a simulation example is proposed to verify the effectiveness of our results.

1. Introduction

Research over synchronization of chaotic systems has drawn much attention in recent years, due to its potential applications in chaotic secure communication systems [1,2]. The methods of synchronization are mainly divided into two categories: continuous methods and discontinuous methods. Continuous methods include adaptive method [3], sliding mode method [4], feedback method [5,6], fuzzy method [7], projective method [6], and so on. Discontinuous methods mainly refer to impulsive method [8,9] and intermittent control method [1,10–12].

Usually, in an intermittent control period, there are two parts, namely “work time” for effectual control and “rest time” for ineffectual control [10]. Furthermore, when the rest time reduces to zero, the intermittent control becomes the usual continuous control, and when the work time reduces to zero, it becomes the impulsive control [10]. Therefore, the continuous control method and the impulsive method are the two extreme state of intermittent control method. As it should be, intermittent control method has been adopted to realize synchronization of chaotic system [1,10–12] and complex neural network [13,14]. In complex neural network, time-delay often exists, which may destroy synchronization [15]. Hence, numerous methods of synchronization of delayed complex neural network has been studied, notably complete synchronization of delayed chaotic neural network [16], synchronization of delayed memristor-based chaotic neural networks [17,18], synchronization of hybrid-coupled delayed dynamical networks [19] and cluster synchronization of delayed heterogeneous dynamical networks [20]. Furthermore, synchronization of delayed chaotic systems were also the research focus [21–25].

Originating from fuzzy mathematics, the fuzzy method was applied to control chaotic system by Takagi and Sugeno (TS model) in 1985 [7]. The fuzzy method is mainly used to approximate the nonlinear part of the chaotic system. Then it has been widely studied due to its easy implementation [26]. Besides, the fuzzy method is combined with other methods of control and synchronization, like adaptive fuzzy synchronization [27], impulsive fuzzy control and synchronization [28,29].

It is worth noting that both the fuzzy method and intermittent control are easy to actualize, and no researcher has studied them so far. So in this paper, we combine fuzzy method and intermittent control to realize synchronization of chaotic system. Based on Lyapunov stability theory, we give the fuzzy intermittent control scheme of chaotic system. Besides, we should mention that our fuzzy intermittent methods for synchronization integrated the advantage of both fuzzy methods and intermittent methods. So our fuzzy intermittent method provided a new and useful method for synchronization of chaotic systems. Therefore, compared with other methods [30–34], our fuzzy intermittent method can be easily designed based on fuzzy mathematics and reach synchronization based on intermittent methods.

The rest of the paper is organized as follows. In Section 2, we give TS fuzzy model of chaotic system and intermittent controller. In Section 3, the fuzzy scheme via intermittent controller is given and analyzed via Lyapunov stability theory. At last, numerical simulation is shown in Section 4.

* Corresponding author.
E-mail address: 6180738@qq.com (Y. Wang).

https://doi.org/10.1016/j.chaos.2017.11.024
0960-0779/

2. Problem description

In this section, we will show the model of chaotic system and the form of fuzzy intermittent control.

Generally, the mathematic model of chaotic system can be described as

$$\dot{x}(t) = Ax(t) + f(x(t)) \tag{1}$$

Where $x(t) \in R^n$ is the state of the chaotic system, $A \in R^{n\times n}$ is the linear parameter matrix and $f(x(t)) \in R^n$ is the nonlinear part of chaotic system.

Because the nonlinear part can be controlled by fuzzy method, then the if-then rule of T-S model is applied to the chaotic system [6] Plant Rule i: if $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2}, $z_3(t)$ is M_{i3}; then

$$\dot{x}(t) = A_i x(t), i = 1, 2, \cdots, r \tag{2}$$

Where $z(t)$ is the observed variables, M_i is the fuzzy set, r is the rule number of "if-then", $A_i \in R^{3\times 3}$ is the fuzzy matrix which is get considering Eq.(2)

To discuss synchronization, we generally let Eq. (1) as the drive system, and the response system as

$$\dot{y}(t) = Ay(t) + f(y(t)) + u(t) \tag{3}$$

Where $y(t) \in R^n$ is the state of the chaotic system(3). And $u(t)$ is intermittent controller which need to design.

Here, the plant rule of response system(3) should be same with the drive system.

HencePlant Rule i: if $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2}, $z_3(t)$ is M_{i3}; then

$$\dot{y}(t) = A_i y(t) + u(t), i = 1, 2, \cdots, r \tag{4}$$

Let the error as $e(t) = y(t) - x(t)$, then $e(t) = (e_1(t), e_2(t), \cdots, e_n(t))^T$. Then combining Eqs. (2) and (4), the error system under plant rule i (where $i = 1, 2, \cdots, r$) as

$$\dot{e}(t) = A_i e(t) + u(t) \tag{5}$$

$u(t)$ is the intermittent control. Without considering fuzzy methods, the intermittent controller is linear control as

$$u(t) = \begin{cases} -ke(t), t \in [\tau_{2m}, \tau_{2m+1}), m = 0, 1, 2, \cdots \\ 0, t \in [\tau_{2m+1}, \tau_{2m+2}), m = 0, 1, 2, \cdots \end{cases} \tag{6}$$

Where $\{t_m\}, m = 0, 1, 2\cdots$ is the set of time points.

However, the intermittent control should also has the fuzzy state. There are two cases: (a) intermittent control $u(t)$ has the same plant rule as drive and response system; (b) intermittent control $u(t)$ has the different plant rule as drive and response system.

Hence, case (a), plant rule of intermittent control is same as drive system and response system.

Plant Rule i: if $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2}, $z_3(t)$ is M_{i3}; then

$$u(t) = \begin{cases} -K_i e(t), t \in [\tau_{2m}, \tau_{2m+1}) \\ 0, t \in [\tau_{2m+1}, \tau_{2m+2}) \end{cases} \tag{7}$$

Where $i = 1, 2\cdots, r$, $m = 0, 1, 2, \cdots$, K_i is the control gain matrix.

Therefore, the error system (5) can be rewritten as

$$\dot{e}(t) = \begin{cases} A_i e(t) - K_i e(t), t \in [\tau_{2m}, \tau_{2m+1}) \\ A_i e(t), t \in [\tau_{2m+1}, \tau_{2m+2}) \end{cases} \tag{8}$$

By using a singleton fuzzifier, product inference, and a center-average defuzzifier, the T-S model of fuzzy control system (8) can be obtained:

$$\dot{e}(t) = \begin{cases} \sum\limits_{i=1}^{r} h_i(z(t))(A_i - K_i)e(t), t \in [\tau_{2m}, \tau_{2m+1}) \\ \sum\limits_{i=1}^{r} h_i(z(t))A_i e(t), t \in [\tau_{2m+1}, \tau_{2m+2}) \end{cases} \tag{9}$$

Where $h_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))}$, $\omega_i(z(t)) = \prod_{k=1}^{r} M_{ik}(z(t))$, $\sum_{i=1}^{r}\omega_i(z(t)) > 0$, $\sum_{i=1}^{r} h_i(z(t)) = 1$, $h_i(z(t)) \geq 0$, $i = 1, 2\cdots, r$. Case (b), plant rule of intermittent control is different from drive system and response system.

Plant Rule i: if $z_1(t)$ is N_{j1}, $z_2(t)$ is N_{j2}, $z_3(t)$ is N_{j3}; then

$$u(t) = \begin{cases} -L_j e(t), t \in [\tau_{2m}, \tau_{2m+1}) \\ 0, t \in [\tau_{2m+1}, \tau_{2m+2}) \end{cases} \tag{10}$$

Where $j = 1, 2\cdots, r$, L_j is the control gain matrix. The output of fuzzy controller is weighted mean value of output of l subsystem, i.e.

$$u(t) = \begin{cases} -\sum\limits_{j=1}^{l} h_j(z(t))L_j e(t), t \in [\tau_{2m}, \tau_{2m+1}) \\ 0, t \in [\tau_{2m+1}, \tau_{2m+2}) \end{cases} \tag{11}$$

Where $h_j(z(t)) = \frac{\omega_j(z(t))}{\sum_{j=1}^{l}\omega_j(z(t))}$, $\omega_j(z(t)) = \prod_{k=1}^{l} N_{jk}(z(t))$, $\sum_{j=1}^{l}\omega_j(z(t)) > 0$, $\sum_{j=1}^{l} h_j(z(t)) = 1$, $h_j(z(t)) \geq 0$, $i = 1, 2\cdots, r$.

When the intermittent controller $u(t)$ is(11), by using a singleton fuzzifier, product inference, and a center-average defuzzifier, the T-S model of fuzzy control system (8) can be obtained:

$$\dot{e}(t) = \begin{cases} \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{l} h_i(z(t))h_j(z(t))\big(A_i - L_j\big)e(t), t \in [\tau_{2m}, \tau_{2m+1}) \\ \sum\limits_{i=1}^{r} h_i(z(t))A_i e(t), t \in [\tau_{2m+1}, \tau_{2m+2}) \end{cases} \tag{12}$$

Where $h_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))}$, $\omega_i(z(t)) = \prod_{k=1}^{r} M_{ik}(z(t))$, $\sum_{i=1}^{r}\omega_i(z(t)) > 0$, $\sum_{i=1}^{r} h_i(z(t)) = 1$, $h_i(z(t)) \geq 0$, $i = 1, 2\cdots, r$.

3. Main result

In this section, we proposed the fuzzy scheme for synchronization of chaotic system via intermittent control. Based on the analysis in Section 2, we propose two theorems corresponding to two cases.

For case (a) i.e. plant rule of intermittent control is the same as drive system and response system. The T-S model of error system is (9). Then we get Theorem 1.

Theorem 1. *Intermittent control $u(t)$ is (7). Let λ_i be the largest eigenvalue of matrix $A_i^T + A_i$, and k_i be the smallest eigenvalue of matrix $K_i + K_i^T$. If the following conditions are true:*

(1) $\lambda - k < 0$, *where* $\lambda = \max_i\{\lambda_i\}$, $k = \min_i\{k_i\}$;
(2) $(\lambda - k)(\tau_{2m+1} - \tau_{2m}) + \lambda(\tau_{2m+2} - \tau_{2m+1}) < 0$, *for* $\forall m \in N$;
(3) *For* $\forall m \in N$, *there exist a positive constant* $\theta_m > 1$, *such that*

$$\theta_m \exp\left((\lambda - k)(\tau_{2m+1} - \tau_{2m}) + \lambda(\tau_{2m+2} - \tau_{2m+1})\right) < 1 \tag{13}$$

Then the error system (9) is asymptotic stability.

Proof: The Lyapunov function is constructed as

$$V(t) = e^T(t)e(t) \tag{14}$$

In the interval $t \in [\tau_{2m}, \tau_{2m+1})$, applied controller. Then the derivative of Lyapunov function $V(t)$ as follow: