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Dynamics of a RLC series circuit with hysteretic iron-core inductor

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ABSTRACT

Inductive devices with ferromagnetic core are widely used in many electronic circuits to store magnetic energy. They should be treated as nonlinear devices, and the nonlinearity of their characteristics arises from the dependence of inductance on current. Such inductors display saturation and hysteresis behaviors. In the present paper, we report a new mathematical model based on the experimental data of hysteresis for ferromagnetic core inductors. We used the model to determine analytically the expression of current in a RLC series circuit forced by an alternating source. Multi periodic and high amplitude chaotic signals are observed and good agreement is found between theoretical and experimental results.

1. Introduction

Ferromagnetic cores are basic constructive elements of transformers and inductors for wide application areas, ranging from analog and digital microelectronics toward power converters and power systems. The role of a magnetic material core in an inductor is to produce by a given applied field, a higher flux compared to the one produced in air, and to serve as magnetic path for the flux. It is known that a rigorous study and design optimization of such electromagnetic devices are difficult because of nonlinearity, electromagnetic inertial behavior and other related phenomena such as saturation, anisotropy, magnetic hysteresis and induced eddy currents.

In general, all electrical circuit elements respond in a nonlinear way to any form of electrical input. While exceptionally simple passive elements such as resistors, capacitors, and air core inductors do respond to a first order approximation nearly linearly, in devices that have ferromagnetic cores, the relationship between flux density and magnetic field strength in the core is nonlinear. This nonlinear relationship depends on several factors among which the chemical constitution and structure of the magnetic material, the technological process for its fabrication, and the way the material is used. The nonlinear characteristic of magnetic materials is irreversible, hence exhibits hysteresis.

Magnetic hysteresis modeling has attracted the interest of researchers since many years. Among the best known methods, one could quote the methods separately describing higher and lower branches of the cycle as well as the rules of passing from one to the other [1,2]. These methods are simple and allow an easy calculator computation [3]. On the other hand, the Preisach model [4] and its derivatives are known as being one of the most powerful techniques currently used to figure out hysteresis in magnetic circuits [5]. Based on the elementary hysterons association, this method uses repartition functions which allow to approximate the hysteresis behavior via analytic [6,7] or numeric methods [8,9].

Another convenient method is the Jiles-Atherton model. Based on physical consideration, it decomposes the total magnetization into a sum of reversible and irreversible components [10]. One of the advantages of this model is its reduced number of parameters (five in classical configuration) but their estimation needs particular experimental tries [11] and makes the identification process difficult [12–14].

Nevertheless, despite the existence of the above mentioned references, we are the view point that there are not enough studies devoted to the determination of parameters to be used in circuit calculations and dimensioning, such as for inductances in all working regions of the involved ferromagnetic materials (linear, intermediate and saturation), geometries and number of turns as performed by the authors of reference [15].

The main objective of this work is to show how both the amplitude of current through a ferromagnetic core inductor and the that of voltages across circuit elements can be predicted. Furthermore,

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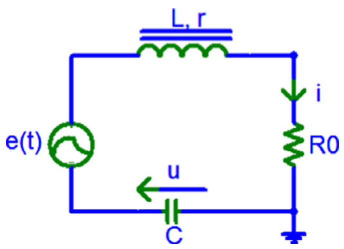

Fig. 1. Circuit diagram modeled in this paper.

we want to produce chaotic signals of high amplitudes using a circuit containing a ferromagnetic core inductor. In fact, high amplitude chaotic signals are needed in communication masking as well as in industrial applications such as electromechanical mixing and sieving, just to name some.

In this paper we shall address the problem of modeling the apparently simple circuit which comprises solely three elements, namely a linear resistor and capacitor in series with a saturating and hysteretic inductor, driven by an alternating voltage source.

The outline of the paper is as follows: in Section 2, the dynamical behavior of the circuit containing a resistor and a ferromagnetic core inductor forced by an alternating generator is analyzed. Next, theoretical and experimental results obtained when the circuit is forced by a sine signal are presented in Section 3. In Section 4, numerical and experimental results in the case of pulse voltage source are given and the paper is concluded with some remarks and future prospects in Section 5.

2. Dynamics of a RLC series circuit containing a ferromagnetic core inductor

The aim of this section is to analyze the dynamics of a RLC series circuit with a ferromagnetic core inductor.

2.1. Circuit and mathematical model

Fig. 1 presents the circuit under study. r is the internal resistance of the inductor and R_0 is an additional resistor needed both to reduce the amplitude of current through the inductor and to measure the current flowing through the circuit.

The inductance of an inductor that contains a ferromagnetic material is described by the following mathematical expression [15]:

$$L = \frac{\mu_0 N^2 A}{\ell} + \frac{B_s N A}{i} \tanh\left(\frac{\alpha N i}{2\ell} - \frac{\sigma}{2}\right), \quad \text{with } \sigma = \beta sign\left(\frac{di}{dt}\right). \tag{1}$$

There are nine parameters appearing in Eq. (1) that are briefly presented below (for more detailed definitions, see [15]). B_s is the saturation flux density, A and ℓ are respectively the cross sectional area and the average length of the magnetic material. N is the number of turns, μ_0 is the permeability of the free space, i is the current through the winding. Parameters α and β are function of the remanence (B_r), the coercive magnetic field (H_c) and the saturation flux density B_s, as mathematically defined below:

$$\alpha = \frac{1}{H_c} \ln\left(\frac{B_s + B_r}{B_s - B_r} \cdot \frac{B_s - \mu_0 H_c}{B_s + \mu_0 H_c}\right) \text{ and } \beta = \ln\left(\frac{B_s + B_r}{B_s - B_r}\right). \tag{2}$$

To analyze the effect of the new parameters on the inductance of the ferromagnetic inductor, we have plotted as shown in Figs. 2a) and 2b) the curves of L as function of the coercive magnetic field (H_c) and of the number of turns N of the coil respectively.

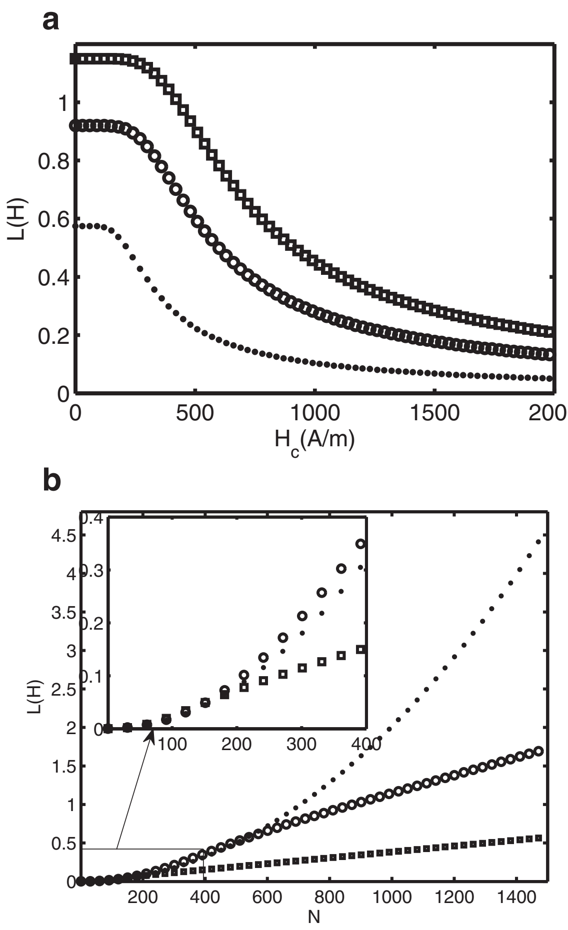

Fig. 2. a) Effect of the coercive magnetic field on L, the curves with dots, circles and squares respectively are plotted for increasing values of $N = 500$, 800 and 1000 turns. b) Effect of the number of turns L: the curves with dots, circles and squares respectively are plotted for the following values of current $i = 10$mA, 200mA and 1A.

In the graph of Fig. 2a), the curves with dots, circles and squares respectively are plotted for increasing number of turns $N = 500$, 800 and 1000. It can be noticed that the inductance of ferromagnetic inductor decreases with the increase of the coercive magnetic field, and increases rather with the number of turns. For Fig. 2b), the same symbols as previously are used to plot the curves for $i = 10$mA, 200mA and 1A respectively. As the curves reveal, the inductance is proportional to N^2 for small amplitudes of current through the inductor. Similar report can be done for small values of N (see the zoom). Meanwhile the relation between L and N becomes approximatively linear for high amplitudes of current and especially for high values of N.

Fig. 3 is considered to analyze the effects of other parameters on the inductance L, such as the applied current plotted in Fig. 3a) and the remanence of the ferromagnetic material (Fig. 3b)).

For Fig. 3a), the curves with dots and squares are plotted respectively for $N = 500$ and 1000 turns. Regarding the shape of the curve, this result is similar to those obtained experimentally by the authors of references [16,17]. As the applied current flowing through an inductor increases, the inductance falls. Similarly, for the graph of Fig. 3b), the curves with dots, with circles and with squares are plotted respectively for $N = 500$, 800 and 1000 turns. Here, the remanence B_r can increase from 0 up to the saturation flux density B_s. It can be noticed from the curves that, all other factors being equal, greater remanence results in greater inductance.