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A plethora of coexisting strange attractors in a simple jerk system with hyperbolic tangent nonlinearity

J. Kengne[a,*], S.M. Njikam[a,b], V.R. Folifack Signing[a,b]

[a] Laboratoire d'Automatique et Informatique Appliquée (LAIA), Department of Electrical Engineering, IUT-FV Bandjoun, University of Dschang (Cameroon)

[b] Laboratory of Electronics and Signal Processing, Department of Physics, University of Dschang.Dschang P.O. Box 67, (Cameroon)

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ABSTRACT

In the present contribution, the dynamics of a simple autonomous jerk system with hyperbolic tangent nonlinearity is considered. The system consists of a linear transformation of Model MO13 previously introduced in [Sprott, 2010]. The form of nonlinearity is interesting in the sense that with the variation of a control parameter, saturation may be approached gradually obeying hyperbolic tangent function, as in the case of magnetization in ferromagnetic system, non ideal op. amplifier, solar-wind-driven magnetosphere-ionosphere system, and activation function in neural network. The fundamental properties of the model are discussed including equilibria and stability, phase portraits, Poincaré sections, bifurcation diagrams and Lyapunov exponents' spectrum. Period doubling bifurcation, antimonotonicity (i.e. concurrent creation an annihilation of periodic orbits), chaos, hysteresis, and coexisting bifurcations are reported. As a major outcome of this paper, a window in the parameter space is revealed in which the jerk system experiences the unusual phenomenon of multiple coexisting attractors (i.e. coexistence of two, four or six disconnected periodic and chaotic self excited attractors) resulting from the simultaneous presence of three families of parallel bifurcation branches and hysteresis. To the best of the authors' knowledge, no example of such a simple and 'elegant' 3D autonomous system capable of six different strange attractors is reported in the relevant literature. Some PSpice simulations based on a physical implementation of the system are carried out to support the theoretical analysis.

1. Introduction

During the past few years, the analysis and design of nonlinear dynamic systems with multiple attractors has been the subject of increasing interest. As an active research topic, it has also advanced significantly due to many contributions from different researchers. Multistability or the occurrence of multiple attractors for the same values of system parameters is a feature inherent in various nonlinear dissipative systems [1–20]. This property is relevant from the point of view of practical applications (as it may result in unexpected and disastrous consequences) and has been found in diverse domains of physical and technological systems such as laser [2], biological system [3], chemical reactions [4], Lorenz system [5], Newton–Leipnik system [6], 3D autonomous systems [7–9], and electrical circuits [10–16], just to name a few. A nonlinear system with a single attractor is called a monostable system while a bistable system has two different attractors. A system with more than two attractors is referred to as a multistable system. A novel and striking form of this phenomenon arises when a system can have an infinite number of coexisting attractors, where each attractor is related with a particular set of initial conditions [17]. In a multistable system, the long term evolution of the system's motion depends crucially on the choice of initial conditions. Different types of attractors can coexist such as fixed points, limit cycles chaotic and hyperchaotic attractors. Also, it is important to reveal possible multistable attractors in the ensemble and define different sets of initial conditions (i.e. basins of attraction) allowing to select between the desired states. The coexistence of multiple attractors is classically studied with the help of bifurcation diagrams obtained by carrying out a smooth sweeping of one of the bifurcation control parameters. However, such an approach is suited for generalized bistability, which highlights zones of hysteresis in the systems responses as a control parameter varies smoothly in different directions. This technique fails to identify isolated (i.e. parallel) branches, which can coexist in the system response and only be reached by a discontinuous jump in initial conditions. Multiple coexisting attractors are found in systems with symmetry [18,19] as well as in those without any symmetry [20]. However systems with symmetry are much more suited to develop multiple coexist-

* Corresponding author.
E-mail address: kengnemozart@yahoo.fr (J. Kengne).

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ing attractors. Accordingly such type of systems are of interest in the sense that they can experience symmetry breaking and offer the possibility that a symmetric pair of attractors will exhibit an attractor-merging crisis as some bifurcation parameter is changed.

Symmetry has always played an important role in physics, from fundamental formulations of basic principles to concrete applications, and is found in a variety of nonlinear and chaotic systems. This work focuses on the dynamic of a symmetric jerk system [21–25] with a single hyperbolic tangent nonlinearity [26–28] paying particular attention on the chaos mechanism as well the occurrence of multiple coexisting attractors. The system is a linear transformation of Model MO13 previously introduced in the excellent textbook of Sprott [22]. First of all, it ought to be stressed that the form of nonlinearity is interesting in the sense that with the variation of a control parameter, saturation may be approached gradually obeying hyperbolic tangent function, as in the case of magnetization in ferromagnetic system, non ideal op. amplifier, solar-wind-driven magnetosphere-ionosphere system, and activation function in neural network, just to name a few. Briefly recall that jerk systems [21–25] are third-order differential equations of the form $\dddot{x} = J(\ddot{x}, \dot{x}, x)$ where the nonlinear function $J(.)$ is called the "jerk," because the successive time derivatives of the displacement in a mechanical system are the velocity, acceleration, and jerk. Owing to the presence of the hyperbolic tangent nonlinearity, this particular jerk system is highly symmetric and thus shows potential to experience multiple coexisting attractors. In retrospect, the occurrence of more than two attractors in jerk dynamic systems was first reported by Kengne and colleagues [10]. The authors used both numerical and experimental methods to demonstrate the possibility of four distinct periodic and chaotic attractors in a cubic jerk system in some ranges of the system parameters. Basins of attraction of various competing attractors were computed showing extremely complex basin boundaries. This interesting property of multiple coexisting attractors was preserved when replacing the cubic nonlinearity by a hyperbolic sine one (electronically implemented with two anti parallel diodes [14]) or when exploiting the intrinsic nonlinearity of a voltage controlled memristor [15]. More interestingly, the coexistence of six different attractors was recently reported [16] in a jerk system with piecewise quadratic nonlinearity thus setting to six the number of coexisting attractors in jerk systems in general. Inspired by the above mentioned results, this work concentrates on the dynamics of a jerk system with hyperbolic tangent function [22,26–30] owing to its theoretical and practical relevance (see above). Also, in order to shed more light on the dynamics of this particular jerk system, our objective in this paper can be summarize as follows: a) to consider the dynamics of the jerk system model MO13 of Sprott [22] and depict the chaotic mechanism; b) to explore and reveal the region in the parameter space corresponding to regimes of multiple coexisting attractors; c) to carry out the simulation of the system in PSPICE (Simulation Program for Integrated Circuit emphasis, PC version) to verify the results of numerical analyses.

The remainder of the paper is arranged as follows. Section 2 introduces the mathematical model of the jerk system under investigation and discusses its basic properties including symmetry, dissipation, fixed points, and local stability. In Section 3, the bifurcation structures of the system are numerically investigated. A window in the parameter space is revealed in which the system exhibits four and six coexisting attractors with different statistical properties. Basins of attraction of various competing attractors are computed revealing extremely complex basins boundaries. In Section 4, a suitable electronic circuit is designed for the analog simulation of the system. PSpice based simulations are carried out to verify the results of theoretical analysis. Finally, Section 5 summarizes and concludes the outcome of the whole work.

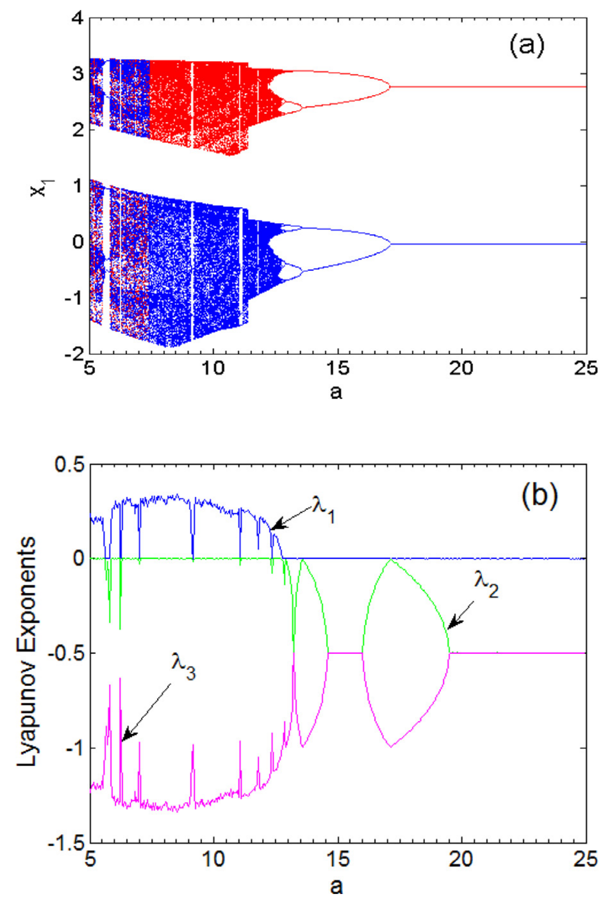

Fig. 1. Bifurcation diagram (a) showing local maxima of the coordinate x_1 versus a and the corresponding graph (b) of Lyapunov exponents ($\lambda_{\max}$) (computer for $\gamma = 1.08, \mu = 1$) plotted in the range $5 \le a \le 25$. A positive value of largest Lyapunov exponent ($\lambda_1 > 0$) indicates chaos while regular dynamics are marked with negative values of largest Lyapunov exponent ($\lambda_2 < 0$).

2. Description and analysis of the model

2.1. The model

The mathematical representation of the jerk system considered in this work is defined by the following set of three coupled first order nonlinear differential equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a x_3 \\ \dot{x}_3 = -3x_1 - \gamma x_2 - \mu x_3 + 6\tanh(x_1) \end{cases} \tag{1}$$

where the dot denotes differentiation with respect to time, and $a, \gamma, \mu \ge 0$ are control parameters. System (1) is obtained by performing a linear transformation of model MO13 previously introduced by Sprott [22]. Notice that only one state variable (namely x_1) is concerned with the hyperbolic nonlinearity of the system. The presence of this nonlinearity accounts for the complex and striking features experienced by the whole system. Equivalently, system (1) can be rewritten in the jerk form as follows:

$$\dddot{x} + \mu\ddot{x} + a\gamma\dot{x} = \varphi(x) \tag{2}$$

where $\varphi(x) = -3ax + 6a\tanh(x)$. Eq. (2) shows that our model is a member of the class of 'elegant' jerk dynamical systems discussed in [22]. More interestingly, system (1) represents one of the simplest autonomous 3D system reported to date, capable of displaying six disconnected chaotic and periodic attractors (see Sect. 3 and 4) depending uniquely on the choice of initial conditions. The