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Dynamical properties and complex anti synchronization with applications to secure communications for a novel chaotic complex nonlinear model

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In this work, we present another chaotic (or hyperchaotic) complex nonlinear framework. This chaotic (or hyperchaotic) complex framework can be considered as a speculation of "Guan" framework [1]. The new framework is a 5-dimensional nonstop real independent chaotic (or hyperchaotic) framework. The fundamental properties and elements of our framework are examined. Once with the all parameters are real and the other with one of these parameters is a complex parameter. When we examine the dynamics of the new framework and the parameters in real form, the conduct of the new framework is chaotic. While when one of these parameters is complex, our new framework is hyperchaotic. On the premise of Lyapunov capacity and dynamic control method, a scheme is designed to accomplish the complex anti-synchronization of two identical chaotic (or hyperchaotic) attractors of these frameworks. The effectiveness of the acquired outcomes will be delineated by a simulation case. Numerical outcomes are schemed to indicate state variables and errors of these chaotic attractors after synchronization to show that synchronization is accomplished. The above outcomes will give hypothetical establishment to the secure communication applications in light of the proposed scheme. In this secure communication scheme, synchronization between transmitter and receiver is accomplished and message signals will be recuperated. The encryption and rebuilding of the signals will be simulated numerically.

1. Introduction

Synchronization of chaos (or hyperchaos) shows a manner wherein two chaotic (or hyperchaos) frameworks (either identical or extraordinary) control a conveyed characteristic for their motion to a standard performance because of a coupling or to a forcing. For more points of interest on synchronization of chaotic frameworks see Refs[2,3]., while for hyperchaotic frameworks, for instance, Refs[4–6]. and references therein. The chaos synchronization phenomenon was discovered in the early 90s [7]. As of late, the search for synchronization has moved to chaotic and hyperchaotic frameworks portrayed by real (or complex) variables. A dynamical framework is called chaotic in the event that it is deterministic, has a long-term aperiodic conduct, and shows sensitive reliance on the initial conditions. On the off chance that the framework has (at least more than one) positive Lyapunov type, then the framework is called chaotic (or hyperchaotic). Be that as it may, the insignificant measurement for a (continuous) hyperchaotic framework is 4 [4]. A chaotic (or hyperchaotic) framework displays sensitive reliance on the initial conditions. This implies two trajectories evolving of two diverse adjustment initial conditions isolate exponentially over the span of the time. Subsequently, chaotic (or hyperchaotic) frameworks naturally challenge synchronization, in light of the fact that even two identical frameworks starting from marginal extraordinary initial conditions would advance in time in an unsynchronized manner. This is an important functional issue since exploratory initial conditions are never known consummately [1].

Since the synchronization phenomenon is exceptionally fascinating and imperative, a great deal of work has been dedicated to studying it. For instance, complete (or full) synchronization [1,7], generalization synchronization [1,8], phase synchronization [9] and lag synchronization (LS) [10] et cetera.

Particularly, there are some new sorts of synchronization that cannot be examined for real dynamical frameworks, for instance,

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module-phase synchronization [11], complex complete synchronization (CCS) [12], complex lag synchronization (CLS) [13], complex anti-lag synchronization (CALS) [14,15], complex projective synchronization (CPS) [16] and complex modified projective synchronization (CMPS) [17]. These new sorts of synchronization are examined for chaotic complex nonlinear frameworks. In complex space, there are two critical amounts of module and phase. Along these lines, the practices of the module and phase are studied in [11–17].

Consider the chaotic (or hyperchaotic) complex nonlinear framework with specific parameters as follows [13,16]:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{H}\mathbf{x} + F(\mathbf{x}, \mathbf{z}), \\ \dot{\mathbf{z}} = g(\mathbf{x}, \bar{\mathbf{x}}, \mathbf{z}), \end{cases} \tag{1}$$

where $\mathbf{x} = (x_1,\ x_2, \ldots,\ x_n)^T$ is a state complex vector, $\mathbf{x} = \mathbf{x}^r + j\mathbf{x}^i$, $\mathbf{x}^r = (x_1^r, x_2^r, \ldots, x_n^r)^T$, $\mathbf{x}^i = (x_1^i, x_2^i, \ldots, x_n^i)^T$, $j = \sqrt{-1}$, T means transpose, $\mathbf{H} \in R^{n\times n}$ is real (or complex) matrix of framework parameters, $F = (f_1,\ f_2, \ldots,\ f_n)^T$ is a vector of nonlinear function, g is a real capacity of linear and nonlinear terms, $\mathbf{z}$ is vector of real variables and superscripts r and i remain for the real and imaginary parts of the state complex vector $\mathbf{x}$.

In 2014, Guan et al. [1] represented the chaotic framework in real form as:

$$\begin{aligned} \dot{x}(t) &= ax(t) - y(t)z(t) - y(t) + k, \\ \dot{y}(t) &= -by(t) + x(t)z(t), \\ \dot{z}(t) &= x(t)y(t) - cz(t), \end{aligned} \tag{2}$$

where $x(t)$, $y(t)$, $z(t)$ are the real state variables and they are functioning in time, however, not unequivocally, a, b, c and k are the parameters of framework (2). The most prominent characteristic of the framework (2) is that it can produce numerous attractors, including various chaotic and different occasional attractors. In addition, it is effectively discovered that this framework additionally can create a four-scroll chaotic attractor.

In this examination, we present a cutting-edge chaotic (or hyperchaotic) show with complex parts as a generalization of the framework (2):

$$\begin{aligned} \dot{x}(t) &= ax(t) - y(t)z(t) - y(t) + k, \\ \dot{y}(t) &= -by(t) + x(t)z(t), \\ \dot{z}(t) &= \frac{1}{2}[(\bar{x}(t)y(t) + x(t)\bar{y}(t)] - cz(t), \end{aligned} \tag{3}$$

where a, b, c are positive parameters, k is a control parameter which can be a real or complex parameter. The variables $x(t) = x^r(t) + jx^i(t)$, $y(t) = y^r(t) + jy^i(t)$ are complex elements, $z(t)$ is real factor, coordinating r and i symbolize the real and imaginary parts.

Remark 1. Most extreme of the chaotic (or hyperchaotic) complex frameworks can be portrayed by (1), for example, chaotic complex Lorenz, Chen and Lü frameworks [2,4,5]. Our framework (3) can be considered as a special instance of (1) where $\mathbf{x} = (x_1, x_2)^T = (x, y)^T$, $\mathbf{z} = z$.

The parameter k is the influential parameter in the conduct of the framework (3). In the event that we consider k as a real parameter, framework (3) is a chaotic framework. In any case, when we select $k = k^r + jk^i$ as a complex parameter, the framework (3) will create the hyperchaotic conduct. In a manner of speaking, the technique for choosing parameters, regardless of whether real or complex influences the conduct of the framework. We would like to clear up this beforehand unexplored impact by studying the conduct of the framework (3). Once when the parameter k is real and the other when it is a complex.

Also, studying a novel kind of synchronization which we can name as complex anti synchronization (CAS). The term of CAS can be managed as synchronizing among AS [18] and CS [19]. AS happens between the real piece of the main framework and the imaginary piece of a slave framework, while CS happens between a real piece of the slave framework and an imaginary piece of the main framework. We would like to recommend a general scheme to consider and accomplish the CAS of two identical chaotic complex nonlinear frameworks in the form (1). With a specific end goal to show the results of our scheme of two identical frameworks of the form (1) we pick, for instance, the chaotic complex nonlinear shows (3).

This paper is sorted out as follows; in the following section, we research the essential properties of the framework (3) once when k is the real parameter and the other when k is complex parameter. In Section 3 the meaning of CAS is familiar and a scheme to accomplish CAS of chaotic (or hyperchaotic) complex nonlinear frameworks is proposed. In the fourth section, we study CAS of two identical chaotic (or hyperchaotic) complex frameworks (3) as an instance of the third portion. An essential application for the secure communication, in light of the eventual outcomes of the CAS, is illustrated. At last, we will locate the principal conclusions of our examinations are completed in Section 5.

2. Basic properties of system (3)

2.1. When k is real parameter

We study the essential dynamical investigation of our new framework (3) when k is real parameters.

The real form of framework (3), for this situation, peruses:

$$\begin{aligned} \dot{x}^r(t) &= ax^r(t) - y^r(t)z(t) - y^r(t) + k, \\ \dot{x}^i(t) &= ax^i(t) - y^i(t)z(t) - y^i(t), \\ \dot{y}^r(t) &= x^r(t)z(t) - by^r(t), \\ \dot{y}^i(t) &= x^i(t)z(t) - by^i(t), \\ \dot{z}(t) &= x^r(t)y^r(t) + x^i(t)y^i(t) - cz(t). \end{aligned} \tag{4}$$

Apparently, framework (4) has six quadratic nonlinear terms and one constant term. The framework (4) has a few chief dynamical characteristics as tracking:

2.1.1. Symmetry and invariance

In this framework (4), we remark that this framework is invariant the transformation:

$(x^r(t), x^i(t), y^r(t), x^i(t), z(t)) \Rightarrow (x^r(t), -x^i(t), y^r(t), -y^i(t), z(t))$, therefor if $(x^r(t),\ x^i(t),\ y^r(t),\ x^i(t),\ z(t))$ is the explication of framework (4), then $(x^r(t),\ -x^i(t),\ y^r(t),\ -x^i(t),\ z(t))$ is known as the solution of the comparable framework.

2.1.2. A dissipative system and existence of the attractor

It is clear to find $\nabla \cdot V = \frac{\partial \dot{x}^r}{\partial x^r} + \frac{\partial \dot{x}^i}{\partial x^i} + \frac{\partial \dot{y}^r}{\partial y^r} + \frac{\partial \dot{y}^i}{\partial yi} + \frac{\partial \dot{z}}{\partial z} = 2a - 2b - c < 0$. At the point when $2a < (2b + c)$, the framework is dissipative and meet to $\frac{dV}{dt} = e^{-(2a-2b-c)t}$ with sort form. It infers that the volume part V_0 contracts to the volume segment $V_0 e^{-(2a-2b-c)t}$ at the time t. Exactly when $t \to \infty$, , the every volume part which contains the framework bearing gathers to 0 with type rate form $2a - 2b - c$. Subsequently, most of the framework headings will, at last, be restricted to zero volume subset, and the dynamic improvement is settled on an attractor.