Chaos, Solitons and Fractals 105 (2017) 38–42

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Disturbances and complexity in volatility time series

Salim Lahmiri [a], Stelios Bekiros [b,c,*]

[a] ESCA School of Management, 7, Abou Youssef El Kindy Street, BD Moulay Youssef, Casablanca, Morocco
[b] European University Institute (EUI), Florence, Italy
[c] Department of Economics, Via delle Fontanelle 18, I-50014, Florence, Italy

ARTICLE INFO

Article history:
Received 3 October 2017
Accepted 7 October 2017

Keywords:
Financial crisis
Volatility
Long memory
Kolmogorov complexity
Shannon entropy

ABSTRACT

Recent works in econophysics have quantitatively shown that the latest global financial crisis has considerably affected nonlinear dynamics in markets worldwide. In the current study, we focus on complexity in volatility time series during pre-crisis, crisis, and post-crisis time periods. In this regard, a large set of international stock and commodity markets as well as economic uncertainty indices is considered in our work. The main finding is that empirical distributions of long memory parameter, Kolmogorov complexity and Shannon entropy, have all varied across pre-crisis, crisis, and post-crisis time periods. In other words, all three complexity measures are informative and suitable in order to characterize nonlinear dynamics in volatility series throughout the examined sample periods. Indeed, it was found that complexity increased during crisis period, yet diminished during the pre-crisis period. Overall, the latest financial crisis has truly affected complexity revealed in the volatility time series of the world major markets.

1. Introduction

In quantitative finance, volatility plays an important role in explaining sharp daily fluctuations in stock prices, determining the probability of bankruptcy, determining the bid-ask spread, hedging portfolio, quantifying risk, and capital allocation [1]. In this regard, interesting studies have been conducted in recent years to examine multifractal in volatility [2,3], cross-correlations between volatility, volatility persistence and stock market integration [4], option pricing using volatility [5], volatility clustering [6], and volatility forecasting [7,8]. A major event in the last ten years is the global financial crisis that affected financial markets worldwide. Indeed, recent works have shown that it has significantly affected and shaped dynamics of capital and commodity market [9,10], co-dynamics in stock markets [11], and linkages across fertilizer markets [12]. Other appealing studies have found that recent global financial crisis affected the degree of asymmetric multi-fractality in the U.S. stock indices [13], the correlation structure of the stocks composing the S&P price index [14], fragility of global financial market network [15], clustering of global foreign exchange markets [16], and global interbank market network connectivity [17]; to name few.

The main purpose of the current work is to study nonlinear dynamics in volatility of a large set of financial, exchange, and commodity markets before, during and after recent world global financial crisis. Indeed, this issue has received a limited interest in econophysics literature. For instance, it was found that US subprime crisis distressed the behaviour of volatility of the sovereign credit default swap index [18], increased volatility of S&P500, KOSPI, and DAX during crisis [19], made volatility of crude oil markets less predictable [20], and increased level of volatility and responses to chocks in fertilizer markets [21].

Therefore, our contributions to the literature are as follows: First, we examine how long memory in volatility of financial and commodity markets was affected by global financial crisis. Second, we aim to look at how complexity in volatility series has been affected during and after crisis. Third, shapes in randomness through time periods are also investigated. In fact, long memory, complexity, and randomness are three different measures borrowed from analysis of nonlinear dynamical systems [22]. Definitely, by examining shapes in self-similarity, complexity, and randomness, a general description of nonlinear dynamics in volatility series can be obtained through different time periods to better understand how global financial crisis influenced volatility of world major markets. Fourth, a large set of various stock, exchange, and commodity markets is considered. Fifth, various robust statistical tests will be applied to check whether fractality, complexity, and randomness in volatility series have been changed across pre-, during, and post-crisis time periods. Sixth, our study will surely enrich econophysics literature on the subject by examining nonlinear dynam-

* Corresponding author at: Department of Economics, Via delle Fontanelle 18, I-50014, Florence, Italy.
E-mail addresses: slahmiri@esca.ma (S. Lahmiri), stelios.bekiros@eui.eu (S. Bekiros).

https://doi.org/10.1016/j.chaos.2017.10.006
0960-0779/

ics of volatility in twenty four international stock markets, three exchange markets, and ten commodity markets. Furthermore, in order to enrich our work, volatility time series in volatility index (VIX) of the Chicago Board Options Exchange (CBOE) and in the United States economic policy uncertainty index (EPU) are also examined. For instance, the former is a gauge of market expectations and investor sentiment used by numerous investors in trading instruments related to the market's expectation of future volatility; and, the latter is a proxy of policy-related economic uncertainty. In fact, it is interesting to study nonlinear dynamics in volatility time series of VIX and EPU in times of crisis to better understand their effects on investor's expectations related to trading instruments and their effects on policy-maker decisions related to business cycle. In short, thirty nine different volatility series are under study in our investigation related to most recent global financial crisis. Indeed, it is worth to provide a framework to significantly better understanding of shifts in nonlinear dynamics of world major financial time series volatilities.

The paper is organized as follows; the Methodology is described in Section 2. Data and empirical results are presented in Section 3. Finally, Section 4 concludes.

2. Methodology

In order to assess nonlinear dynamics in volatility time series, we basically rely on estimating three complexity measures; namely long range parameter denoted d estimated by fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) process [23], Kolmogorov complexity (*KC*) [24], and Shannon entropy (*SE*) [25]. For instance, long range parameter d is used to quantify long-memory in volatility of the underlying time series, *KC* is appropriate to capture the amount of information in an object (or time series) by measuring the length of a description of that object [24], and *SE* is appropriate to assess randomness in a given signal. All these estimates are in fact used to describe complexity in volatility time series. They are described next.

2.1. The FIGARCH model

Assume that the conditional variance of a given time series obeys the following classical generalized autoregressive conditional heteroskedasticity (GARCH) model [26]:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \tag{1}$$

where $\varepsilon_t = y_t - E_{t-1}\,[y_t]$ is the prediction error of the time series y, $E[\cdot]$ is the expectation operator, σ is the conditional variance, $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, L is the lag operator, and:

$$\alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_q L^q \tag{2}$$

$$\beta(L) \equiv \beta_1 L + \beta_2 L^2 + \cdots + \beta_p L^p \tag{3}$$

Whilst GARCH(p,q) is appropriate to describe short-memory in volatility of time series, the FIGARCH(p,d,q) model [23] was initiated to gauge long memory in volatility. Especially, the FIGARCH(p,d,q) model is given by:

$$\phi(L)(1-L)^d\varepsilon_t^2 \equiv \omega + [1-\beta(L)]\nu_t \tag{4}$$

where, $\nu_t = \varepsilon_t^2 - \sigma_t^2$ and $0 < d < 1$ is the fractional difference (long range) parameter used to capture long memory in volatility series. In our study, the FIGARCH(1,d,1) model is estimated by using the quasi-maximum likelihood method as proposed in [27]. It is worth noting that FIGARCH(1,d,1) is a popular statistical process used to estimate long memory in volatility time series as it is valuable and flexible model [28,29].

2.2. Kolmogorov complexity

The Kolmogorov complexity (*KC*) [24] also known as the algorithmic complexity is a non-probabilistic measure used to quantity the amount of information contained in a given time series. In particular, *KC* is basically the length of the shortest algorithm capable of reproducing the underlying time series. Lempel and Ziv [30] proposed an algorithm to calculate *KC*. For instance, let consider a time series $\{x_t\}_{t=1}^n$. Then, a sequence $\{s(i)\}_{i=1}^n$of strings 0 and 1 is constructed where $s(i) = 0$ when $x_i < x^*$, $s(i) = 1$ when $x_i \geq x^*$, and x^* is a threshold. Then, $KC(n)$ is given by:

$$KC(n) = c(n)\frac{\log(n)}{n} \tag{5}$$

where $c(n)$ is the minimum number of distinct patterns contained in sequence $s(i)$ of length n. Asymptotically, $0 \leq KC(n) \leq 1$. In this regard, values of 0 and 1 indicate respectively regular and random series. In the current work, threshold used to construct binary symbol sequences is the mean of the time series.

2.3. Shannon entropy

Shannon entropy [25] is denoted *SE* in this work and it is employed in order to measure the degree of randomness in volatility time series. Assuming a time series $\{x_t\}_{t=1}^n$, the Shannon entropy is expressed as follows:

$$SE(x) = -\sum\nolimits_{y=1}^{n} p_i \log(p_i) \tag{6}$$

where p_i is a discrete probability such that $\sum_i p_i = 1$. For instance, the Shannon entropy (*SE*) reach maximum when all values of the underlying time series time series $\{x_t\}_{t=1}^n$are equally probable. In this regard, when SE approaches log(n), then, the underlying time series is nearly random. In contrary, *SE* reached minimum when a single x_i is certain to occur. For example $Prob(x_i) = 1$.

Finally, after having estimated long memory parameter d, Kolmogorov complexity (*KC*), and Shannon entropy (*SE*) of each variance time series, two-sample t-test [31] and two-sample Kruskal–Wallis (*K-W*) test [32] – are used respectively to test the null hypothesis of equality of the means and null hypothesis of equality of the medians - are applied to each pair of estimated populations of complexity measures through time periods. For instance, the goal is to check whether estimated parameters d in pre-crisis period are statistically different from those in crisis time period. Similarly, one can test whether estimated parameters d in crisis time period are statistically different from those in post-crisis time period. Equally, we also test whether estimated parameters d in pre-crisis time period are statistically different from those in post-crisis time period. In fact, the purpose of statistical test is to verify if the populations of estimated complexity measures have been shifted trough pre-, during, and post-crisis time periods. All previous statistical tests will be performed at 5% statistical significance level.

3. Data and empirical results

We use daily data covering the time period from 22 August 2003 to 1 March 2007 (pre-crisis), 2 March 2007 to 21 September 2012 (crisis), and 24 September 2012 to 17 May 2017 (post-crisis). The data includes twenty four stock markets, three exchanges markets, ten commodity markets, and two financial and economic uncertainty indices, all obtained from Datastream international. International stock markets include NYSE of USA, FTSE of UK, CAC40 of France, DAX30 of Germany, Nikkei225 of Japan, S&P/TSX of Canada, Hang Seng of China, IBEX35 of Spain, ATHEX of Greece, BEL20 of Belgium, SWITZ of Switzerland, PSI of Portugal, MIB of Italy, TAIEX of Taiwan, S&P/ASX 200 of Australia, ISEQ of Ireland, S&P BSE of