Chaos, Solitons and Fractals 105 (2017) 60–68

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

A stochastic SIRS epidemic model incorporating media coverage and driven by Lévy noise

Badr-eddine Berrhazi[a], Mohamed El Fatini[a,*], Aziz Laaribi[b], Roger Pettersson[c], Regragui Taki[a]

[a] Ibn Tofail University, FS, Department of Mathematics, BP 133, Kénitra, Morocco
[b] Hassan II University, FS, Department of Mathematics, Casablanca Morocco
[c] Linnaeus University, Department of Mathematics, 351 95 Växjö, Sweden

ARTICLE INFO

Article history:
Received 24 May 2017
Revised 6 October 2017
Accepted 7 October 2017

MSC:
92B05
60G51
60G57

Keywords:
Stochastic process
Lévy process
White noise

ABSTRACT

In this paper, we establish the existence of a unique global positive solution for a stochastic epidemic model, incorporating media coverage and driven by Lévy noise. We also investigate the dynamic properties of the solution around both disease-free and endemic equilibria points of the deterministic model. Furthermore, we present some numerical results to support the theoretical work.

1. Introduction

One of the deadliest threats to human lives is infectious diseases, in fact they cause more deaths than cancer. Understanding the way of the spread of those diseases is very important in the fight against the disease itself [3,4,7,30]. Media coverage is one of the most used tools to try to control epidemic spreading [35]. There is much evidence that media coverage can play an appreciable role in the spread and control of infectious diseases [13,14,23,25,26,29]. It plays an important role in helping the government authority to make interventions to prevent the disease [8,9]. When an epidemic begins its propagation in a country, the departments of health and of disease control and prevention take necessary means to prevent the disease to spread wildly in the population. One of the measures taken by the government in this matter is to teach the people to act appropriately in these cases using eduction and media [13,14,23,25,32,38]. Mass media have the potential to influence the behavior of the population, they are usually used to deliver preventive health messages in order to take precautions and to avoid negative behavior due to panic and also to present recent information about the disease. In fact, several surveillance organisms rely on the internet and news media to detect upcoming epidemic threats [34]. Recently, various mathematical models have been used to investigate the impact of the media coverage. Cui et al. [23], Tchuenche et al. [36], and Sun et al. [34] used deterministic models to investigate the effects of media coverage on the transmission dynamics.

In this paper, we are interested in a SIRS epidemic model [10,21,37]. The population is divided into three compartments, depending on the epidemiological status of individuals: susceptible (S), infectious (I) or recovered (R). The susceptible population is increased by recruitment of individuals or by loss of immunity of recovered individuals, and reduced by infection of susceptible individuals or natural death. The population of infected individuals is increased by infection of susceptible and diminished by natural death and recovery from the disease. The recovery class is increased by individuals recovering from their infection and is decreased as individuals succumb to natural death or loss of immunity. In the absence of media effect, we assume the incidence rate to be mass action incidence with bilinear interaction given by $\beta_1 SI$, where β_1 is the probability of transmission per contact. However some factors such as media coverage manner of life and density of population may affect the incidence rate. When media coverage is present, social distancing mechanisms come into effect.

* Corresponding author.
E-mail address: melfatini@gmail.com (M. El Fatini).

https://doi.org/10.1016/j.chaos.2017.10.007
0960-0779/

The reporting by media is assumed to be an increasing function of the number of infectious cases, and as a consequence, the contact rate between susceptible and infectious individuals is a decreasing function of the number of infectious cases. Hence, we consider the following nonlinear incidence which reflects some characters of media coverage

$$g(S,I) = (\beta_1 - \frac{\beta_2 I}{\alpha + I})SI,$$

where β_1, is the maximal effective contact rate before media alert, β_2 is the maximal effective contact rate due to mass media alert in the presence of infective and the half saturation $\alpha > 0$ reflects the reactive velocity of individuals and media coverage to epidemic disease. The resulting deterministic SIRS epidemic model incorporating media coverage can be modelled as follows:

$$\begin{aligned} dS &= [\Lambda - (\beta_1 - \frac{\beta_2 I}{\alpha + I})SI - \mu S + \lambda R]dt, \\ dI &= [(\beta_1 - \frac{\beta_2 I}{\alpha + I})SI - (\mu + \gamma)I]dt, \\ dR &= [\gamma I - (\mu + \lambda)R]dt, \end{aligned} \tag{1.1}$$

where, the real positive parameters Λ, μ, γ, λ, have the following features: Λ is the total number of the susceptible, μ represents the natural death rate, γ represents the rate of recovery from infection and λ is the rate of temporary immunity. The function $\frac{I}{\alpha+I}$ is a continuous bounded function that takes into account disease saturation or psychological effects. The terms $\frac{\beta_2 I}{\alpha+I}$ measures the effect of reduction of the contact rate when infectious individuals are reported in the media. Because the coverage report can slow but cannot prevent disease from spreading completely, we have $\beta_1 \geq \beta_2 > 0$. The basic reproduction number [12,15] for this model is given by

$$\mathcal{R}_0 = \frac{\beta_1 \Lambda}{\mu(\mu + \gamma)}.$$

It is a threshold quantity which determines whether an epidemic occurs or the disease simply dies out. In the few past years, epidemic models with stochastic perturbation [16,17,19,22,24,45] have emerged as an interesting topic. Among authors who studied stochastic models with white noise incorporating media coverage we cite Cai et al. [11], where effects of stochastic dynamics of an SIS model incorporating media coverage were investigated. Lui and Zheng [27] investigated the stochastic disease dynamics of an SIS epidemic model on two patches incorporating media coverage. Zhao et al. [44] studied the basic dynamical features of a stochastic SIR epidemic model incorporating media coverage. All these models were perturbed by white noise. However, the jumps play a significant role in evolution of many real dynamical processes [39–43], including the case of epidemic spreading like when encountered with massive diseases like avian influenza. In this paper we are interested in a stochastic SIRS epidemic model incorporating media with a more general perturbation, as following

$$\begin{aligned} dS &= \left[\Lambda - \left(\beta_1 - \frac{\beta_2 I}{\alpha + I}\right)SI - \mu S + \lambda R\right]dt + \sigma_1 S dW_1 \\ &\quad + \int_Y q_1(y)S(t-)\tilde{N}(dt,dy), \\ dI &= \left[\left(\beta_1 - \frac{\beta_2 I}{\alpha + I}\right)SI - (\mu + \gamma)I\right]dt + \sigma_2 I dW_2 \\ &\quad + \int_Y q_2(y)I(t-)\tilde{N}(dt,dy), \\ dR &= [\gamma I - (\mu + \lambda)R]dt + \sigma_3 R dW_3 + \int_Y q_3(y)R(t-)\tilde{N}(dt,dy), \end{aligned} \tag{1.2}$$

where $Wi(t)$ are independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with the filtration $(\mathcal{F}_t)_{t\geq 0}$, satisfying the usual conditions, $X(t-)$ is the left limit of $X(t)$, $N(dt, dy)$ is a Poisson counting measure with the stationary compensator $\nu(dy)dt$, $\tilde{N}(dt,dy) = N(dt,dy) - \nu(dy)dt$ and ν is defined on a measurable subset Y of $[0, \infty)$ with $\nu(Y) < \infty$ and $\sigma_i \geq 0$ represent the intensities of $W_i(t)$, $q_i(y) > -1$, $i = 1, 2, 3$. Our study will be as follow: in the second section, we study the existence and uniqueness of the global positive solution to model (1.2), the third section is devoted to studying the behavior of the solution to the system (1.2) around the disease-free equilibrium E_0, in the fourth section we study the behavior of the solution to (1.2) around the endemic equilibrium E^* and, in the final part, we will present some numerical results supported by real scenarios.

Throughout this paper, we define the operator L associated with the following 3-dimensional stochastic differential equation (SDE)

$$dX = \phi(t, X(t))dt + \psi(t, X(t))dW_t + \int_Y H(t,y)\tilde{N}(dt,dy),$$

by

$$\begin{aligned} LX(t-) &= \sum_{i=1}^{3} \frac{\partial X(t-)}{\partial x^i}\phi^i(t,X) + \frac{1}{2}\sum_{i,j=1}^{3} \frac{\partial^2 X(t-)}{\partial x^i \partial x^j}[\psi^T(t,X)\psi(t,X)]_{ij} \\ &\quad + \int_Y \left[(X(t-) + H(t,y)) - X(t-) - \frac{\partial X(t-)}{\partial x^i}H(t,y)\right]\nu(dy), \end{aligned}$$

where $X = (x^1, x^2, x^3)$. If L acts on a function $F \in \mathcal{C}^{1,2}(\mathbb{R}_+ \times \mathbb{R}^3)$, then

$$\begin{aligned} LF(X(t)) &= F_x(X(t-))\phi(t,X) \\ &\quad + \frac{1}{2}trace(\psi^T(t,X(t-))F_{xx}\psi(t,X(t-)) \\ &\quad + \int_Y F(X(t-) + H(t,y)) - F(X(t-)) \\ &\quad - F_x(X(t))H(t,y)\nu(dy), \end{aligned}$$

where

$$F_x = \left(\frac{\partial F}{\partial x^1}, \frac{\partial F}{\partial x^2}, \frac{\partial F}{\partial x^3}\right), \; F_{xx} = \left(\frac{\partial^2 F}{\partial x^i \partial x^j}\right)_{3\times 3}$$

Then by ItÃ's formula we obtain

$$\begin{aligned} dF(X(t)) &= LF(X(t-))dt + F_x(X(t-))\psi(t,X)dW_s \\ &\quad + \int_Y [F(X(t-) + H(t,y)) - F(X(t-))]\tilde{N}(ds,dy), \end{aligned}$$

Remark 1.1. For the Itô formula for semimartingales with jumps we refer to [5,31,33] for more details.

2. Global positive solution of the system (1.2)

The next theorem ensures the existence and uniqueness of the global positive solution.

Theorem 2.1. *For any given initial value* $(S(0), I(0), R(0)) \in \mathbb{R}^3_+$, *then the model (1.2) has a unique global solution* $(S(t), I(t), R(t)) \in \mathbb{R}^3_+$ *for all* $t \geq 0$ *almost surely.*

Proof. The drift and the diffusion being locally Lipschitz, then for any given initial value $(S(0), I(0), R(0)) \in \mathbb{R}^3_+$, there is a unique local solution $(S(t), I(t), R(t))$ for $t \in [0, \tau_e)$, where τ_e is the explosion time. To show that this solution is global, we need to show that $\tau_e = \infty$ a.s. At first, we prove that $S(t)$, $I(t)$, $R(t)$ do not explode to infinity in a finite time. Let $m_0 > 0$ be sufficiently large so that $S(0)$, $I(0)$, $R(0)$ lies within the interval $[\frac{1}{m_0}, m_0]$. For each integer $m \geq m_0$, we define the stopping time:

$$\begin{aligned} \tau_m = \inf\Big\{ & t \in [0, \tau_e)/S(t) \notin \left(\frac{1}{m}, m\right) \; or \; I(t) \notin \left(\frac{1}{m}, m\right) \\ & or \; R(t) \notin \left(\frac{1}{m}, m\right)\Big\}, \end{aligned}$$