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# Nonlinear dynamics behind the seismic cycle: One-dimensional phenomenological modeling

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ABSTRACT

In present paper, authors examine the dynamics of a spring-slider model, considered as a phenomenological setup of a geological fault motion. Research is based on an assumption of delayed interaction between the two blocks, which is an idea that dates back to original Burridge–Knopoff model. In contrast to this first model, group of blocks on each side of transmission zone (with delayed interaction) is replaced by a single block. Results obtained indicate predominant impact of the introduced time delay, whose decrease leads to transition from steady state or aseismic creep to seismic regime, where each part of the seismic cycle (co-seismic, post-seismic and inter-seismic) could be recognized. In particular, for coupling strength of order $10^2$ observed system exhibit inverse Andronov–Hopf bifurcation for very small value of time delay, $\tau\approx0.01$, when long-period (T = 12) and high-amplitude oscillations occur. Further increase of time delay, of order $10^{-1}$, induces an occurrence of a direct Andronov–Hopf bifurcation, with short-period (T = 0.5) oscillations of approximately ten times smaller amplitude. This reduction in time delay could be the consequence of the increase of temperature due to frictional heating, or due to decrease of pressure which follows the sudden movement along the fault. Analysis is conducted for the parameter values consistent with previous laboratory findings and geological observations relevant from the seismological viewpoint.



## 1. Introduction

It is generally considered that process of accumulation and release of stress along the seismogenic faults always obeys the same rule: period with no movement along the fault (or with aseismic creep), when the stress is being accumulated, is followed by its sudden release, which could be further succeeded by the partial emission of the remained stored energy. These three periods, formally known as inter-seismic, co-seismic and post-seismic, respectively, constitute a single seismic cycle, which could be manifested at regular time intervals (for the strongest seismic events), or, more likely, occurrence of seismic events appears as a random process following Poisson distribution [1]. From the seismological viewpoint previous studies on properties of a seismic cycle resulted in sufficiently accurate characterization of each of the aforementioned periods. It is well known that inter-seismic deformation indicates depth of the zone that will eventually rupture seismically [2] and the rate at which stress is accumulating along the fault zone [3]. The very end of this inter-seismic period could be marked by the occurrence of foreshocks as small partial releases of the stored potential energy before the main event. On the other hand, post-seismic deformation is usually driven by the preceding co-seismic stress change [3] and it could be as large as the fault slip during the main seismic event. Observed post-seismic behavior includes poroelastic deformation [4], frictional afterslip [5] and viscoelastic relaxation [6]. Similarly to the inter-seismicposteriod, post-seismic part of the seismic cycle could be marked by the occurrence of aftershocks, as sudden releases of the remaining stored energy with significantly smaller magnitude in comparison to the main seismic event.

From the purely mechanical viewpoint, it is commonly considered that alternation of seismic cycles could be described by irregular stick-slip behavior [7]. For a simple frictional system, like commonly used spring-block model, the occurrence of stick-slip is due to a difference in static and kinetic friction, i.e. once the block starts to slide the friction drops suddenly to a lower level [8]. It is generally considered that surface roughness and normal

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stress level play main role in "pushing" the spring-block model into stick-slip regime [9]. In present analysis, we analyze only the effect of friction on dynamics of spring-block model, by assuming some small constant value of normal stress which does not significantly affect the dynamics of the model. This could correspond to shallow parts of the Earth's crust, or parts where horizontal stresses are much higher that vertical ones, due to significant effect of tectonics and surface erosion which reduced the thickness of the overlying layers.

Results of the pioneer work of Burridge and Knopoff [10] on dynamics of a simple spring-block model set a solid base for succeeding laboratory and theoretical research of seismogenic fault motion. The main outcome of their work is that distribution of displacement sums (i.e. earthquake magnitudes) follows two key macrosesimologic laws: Gutenberg–Richter and Omori–Utsu power law distribution. This finding enabled succeeding researchers a wide specter of additional analyzes, from the purely seismological [11,12], across the tribological [13,14] to purely dynamical [15]. These "dynamical" research are primarily in our focus, since they showed that for a certain parameter range, dynamics of spring-block models exhibit a regular transition between different dynamical regimes, with the eventual occurrence of chaotic dynamics [16,17]. Nevertheless, former studies did not treat the problem of seismic cycle *per se*, except from our previous paper, where we analyzed the impact of transient seismic wave on the dynamics of spring-block model, which resulted in transition between different seismic cycles [18]. One of the goals of the present analysis is to match different dynamical regimes of a spring-block model to appropriate phases of seismic cycles. In particular, the performed analysis should provide answers to the following questions: (1) what are the relevant parameter ranges for which the dynamic of the spring-block model enters the stick-slip regime, (2) what are the main dynamical features of that regime and (3) what does it mean for the real conditions in Earth's crust. In that way, we will be able to reveal the main controlling mechanism behind the regularity of seismic cycle. One should note that, besides seismology, nonlinear models in general have been successively applied in other areas of natural sciences, as well [19–24].

Besides the analogy with the macroseismological laws, another important outcome of the original work of Burridge and Knopoff concerns a delayed transition of motion among two sets of blocks, indicating possible highly complex dynamical behavior. In particular, they showed that displacement among two boundary group of blocks in an one-dimensional chain is being transmitted with a certain time delay, whose order of unit corresponds to the viscosity of the middle set of blocks. Although this finding opened a lot of possibilities for investigating the cause and consequences of such a feature, it was not taken into consideration in succeeding studies. Effect of time delay was previously only implicitly introduced in friction term [25,26], and between the neighboring blocks in an one-dimensional chain of blocks with rate-dependent friction law [27]. In present paper, we analyze the transition between different seismic cycles considering the delayed interaction among the blocks with a rate-and state-dependent friction law. In contrast to our previous work, delayed interaction is assumed between the blocks exhibiting rate-and state-dependent friction law, which corresponds well to the laboratory observations of rock friction. Also, present analysis is conducted for the values of parameters which are either observed in reality or in laboratory conditions. We consider that this behavior is also relevant from the viewpoint of seismology, since different friction conditions along the fault (e.g. different thickness and physico-mechanical properties of fault gouge, impact of pore fluid, etc.) could cause a delayed transition of motion among different parts of the active seismogenic fault.

To sum up, the main idea of the present study is to determine the main dynamical mechanism by which the fault motion model reaches stick-slip like oscillations, as an appropriate dynamical state of a seismic fault motion which includes the interseismic, co-seismic and post-seismic regime. Thereby, dynamics of the relevant model is examined for the parameter values meaningful from the viewpoint of seismology, under the influence of the assumed delayed interaction of variable strength. Introduction of new influential parameters is motivated by the previous laboratory findings, with the aim of modeling the effect of changeable friction properties along the fault. The analysis is conducted using both analytical and numerical methods, former of which involved the application of local bifurcation analysis for the model with constant time delay whose results are corroborated numerically.

## 2. Model development

### 2.1. Original model of fault motion

Our numerical simulations of a spring-block model are based on the system of equations coupled with Dieterich–Ruina rate-and state-dependent friction law [16]:

$$\begin{aligned}\dot{\theta} &= -\left(\frac{\nu}{L}\right)\left(\theta + B\log\left(\frac{\nu}{\nu_0}\right)\right)\\ \dot{u} &= \nu - \nu_0\\ \dot{\nu} &= \left(-\frac{1}{M}\right)\left(ku + \theta + A\log\left(\frac{\nu}{\nu_0}\right)\right)\end{aligned} \tag{1}$$

where parameter $M$ is the mass of the block and the spring stiffness $k$ corresponds to the linear elastic properties of the rock mass surrounding the fault [28]. According to Dieterich and Kilgore [29] the parameter $L$ corresponds to the critical sliding distance necessary to replace the population of asperity contacts. The parameters $A$ and $B$ are empirical constants, which depend on material properties. Variables $u$ and $\nu$ represent displacement and velocity, while $\theta$ denotes the state variable describing the state of the rough surface along which blocks are moving [30]. Parameter $V_0$ represents the constant background velocity of the upper plate Fig. 1). For convenience, system ((2) is non-dimensionalized by defining the new variables $\theta'$, $v'$, $u'$ and $t'$ in the following way: $\theta = A\theta'$, $v = v_0 v'$, $u = Lu'$, $t = (L/v_0)t'$, after which we return to the use of $\theta$, $v$, $u$ and $t$. This non-dimensionalization puts the system into the following form:

$$\begin{aligned}\dot{\theta} &= -\nu(\theta + (1+\varepsilon)\log(\nu))\\ \dot{u} &= \nu - 1\\ \dot{\nu} &= -\gamma^2[u + (1/\xi)(\theta + \log(\nu))]\end{aligned} \tag{2}$$

where $\varepsilon = (B-A)/A$ measures the sensitivity of the velocity relaxation, $\xi = (kL)/A$ is the nondimensional spring constant, and $\gamma = (k/M)^{1/2}(L/v_0)$ is the nondimensional frequency [16]. As it was previously shown [18], a supercritical direct Andronov–Hopf bifurcation curve occurs for the following parameter values $\varepsilon = 0.27$, $\xi = 0.5$ and $\gamma = 0.8$, leading from equilibrium state to regular periodic oscillations.

### 2.2. Fault motion model under study

We analyze the dynamics of two coupled blocks Fig. 1), whose motion is governed by the following system of first-order ordinary