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Control and synchronization of chaos with sliding mode control based on cubic reaching rule

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ABSTRACT

In this study, a new reaching rule in the Sliding Mode Control (SMC) is proposed for chaos control and synchronization. It is applied on the nonlinear Chua's circuit. The SMC signals are provided with the Lyapunov stability theory. Classical, cubic and partial cubic variants of SMC signals are constructed. Numerical simulations are performed to compare the effectiveness of the SMC signals and the results are discussed.

1. Introduction

Chaos control is suppressing the chaotic motions and stabilizing towards an equilibrium point. Control of chaotic systems is applied when chaos is undesirable. Chaos synchronization is matching the state vectors of either two identical chaotic systems which start from different initial values or two non-identical chaotic systems. Synchronization of chaotic systems is useful in encryption, secure communication, etc. Researchers have become interested in controlling and synchronizing chaotic systems after the pioneering efforts of Ott, Grebogi and Yorke on chaos control [1] and Pecora and Carroll on chaos synchronization [2] in 1990. Thus, they become significant issues on the control engineering field. At present, some effective methods are using for the control and synchronization of chaotic systems such as active control [3], SMC [4–9], adaptive control [10], linear feedback control [11], nonlinear feedback control [12], time-delay feedback control [13], passive control [14], backstepping design [15], predictive control [16], and impulsive control [17]. Among these control methods, the SMC theory is often preferred by many researchers in chaos control and synchronization because of the reachability conditions which can be defined in different ways. It can be described as a nonlinear control technique that tries to hold the response of the variable structure systems on a sliding surface by switching the signals. It is characterized by having the ability to control uncertainty in a system with good dynamic characteristics, less control and synchronization time, robustness and insensitiveness to external variations, among other attractive benefits. More detailed information about the theory of SMC method can be reached from many papers in the literature [4–9].

Electronic circuits are linear or nonlinear characteristics paradigms. In the last two decades, researchers have proved that almost all of the electronic and electric circuits display chaotic behaviors in specific conditions linked to the initial values, input signals and chosen parameters. Electronic circuit analysis and mathematical evaluation are fairly difficult as opposed to solving nonlinear differential equations. In this regard, Chua's circuit [18] is one of the famous chaotic attractors. It is a simple electronic circuit. There are also some different versions called modified Chua's circuits [19–21], cubic Chua oscillator [22], *n*-scroll Chua's circuit [23], hyperchaotic Chua systems [24–26], etc. Furthermore, there are plenty of papers investigating the control of Chua chaotic systems. Linear feedback control [27], nonlinear control [28,29], impulsive control [30], adaptive control [31], and SMC [32–37] methods are applied for the control of Chua chaotic systems. SMC signals are used for both the original Chua's circuit [32–35] and the other versions [36, 37]. Besides this, the synchronization of the original Chua's circuit is implemented via piecewise linear coupling [38], impulsive control [30,39], and nonlinear control [40] methods.

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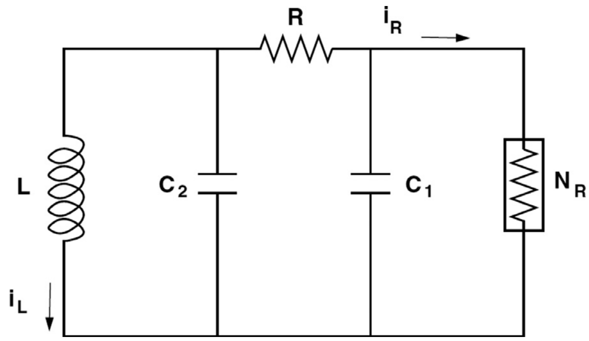

Fig. 1. The Chua's circuit.

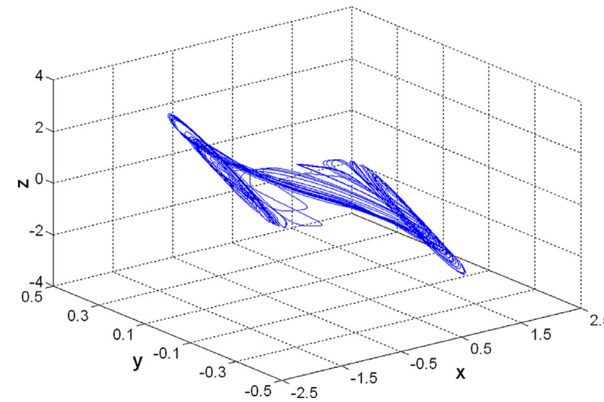

Fig. 2. The Chua chaotic system in three-dimensional phase plane.

Some sliding mode variants such as proportional-integral based adaptive SMC [41], integral SMC [42], and active SMC [43] methods are also used for the original Chua's circuit. However, the synchronization of Chua oscillators is achieved through the SMC method [44,45], but all of them deal with the modified versions of Chua attractors.

In this study, focusing on the Chua's circuit, its chaotic behavior is described; its control and synchronization are applied according to the solutions of nonlinear differential equations through the SMC method. Cubic and partial cubic SMC signals are proposed as a novel access rule. The aim of this study is to show that the proposed cubic reaching rule can be more effective in the SMC for controlling and synchronizing the chaos. The well-known Chua chaotic system is preferred in the simulations. The rest of this paper is arranged as follows: A brief explanation of Chua's circuit and its differential equations are provided in Section 2. Then, its control and synchronization are applied by using SMC method with the classical and cubic SMC signal variants in SectionS 3 and 4, respectively. After, numerical simulations are demonstrated in Section 5. Finally, conclusions are presented in Section 6.

2. Definition of Chua's circuit

The Chua's circuit is one of the well-known simple physical systems which displays chaotic phenomena. In Fig. 1, the Chua's circuit is shown. It is composed by an inductor, a linear resistor, two capacitors, and a three-segment nonlinear resistor [18]. The nonlinear resistor is also called as Chua's diode.

The dimensionless form of the Chua's system is described as [18]:

$$\begin{cases} \dot{x} = \alpha(y - x - f(x)), \\ \dot{y} = x - y + z, \\ \dot{z} = -\beta y, \end{cases} \tag{1}$$

where x, y, and z are the state variables which represent the voltages across the capacitors C_1 and C_2, and the intensity of electrical current in the inductor L, respectively. $\alpha > 0$ and $\beta > 0$ are the constant parameters that are defined by the certain values of the circuit elements. The electrical response of the nonlinear resistor element is described by $f(x)$ function. Its value depends on the particular configuration of the components. It is generally defined as

$$f(x) = bx + \frac{1}{2}(a - b)(\,|x+1| - |x-1|\,), \tag{2}$$

where a and b are real constants. For a specific value of the parameters such as $\alpha = 15.6, \beta = 25.58$, $a = -8/7$, and $b = -5/7$ with the initial value (0, 0, 0.6), the Lyapunov exponents for system (1) are $\lambda_1 \approx 0.3271$, $\lambda_2 \approx 0$, and $\lambda_3 \approx 2.5197$ [18], which makes the Chua's circuit chaotic. Its Kaplan–Yorke dimension is $D_{KY} \approx 2.1298$ [46]. The three-dimensional phase plane of Chua chaotic system is shown in Fig. 2. Due to its shape in the (x, y, z) space, Chua chaotic system is also known as "double scroll".

The equilibrium points of Chua chaotic system can be found by assuming $\dot{x} = 0$, $\dot{y} = 0$, and $\dot{z} = 0$, and solving the equations of system (1) as in the following:

$$\begin{cases} \alpha(y - x - bx - 0.5(a - b)(\,|x+1| - |x-1|\,)) = 0, \\ x - y + z = 0, \\ -\beta y = 0. \end{cases} \tag{3}$$

Therefore, the Chua chaotic system has only one equilibrium point: E_0 (0, 0, 0).

3. Control

The Chua chaotic system defined by Eq. (1) can be rewritten by adding the control signals as follows:

$$\begin{cases} \dot{x} = \alpha(y - x - f(x)) + u_1, \\ \dot{y} = x - y + z, \\ \dot{z} = -\beta y + u_2, \end{cases} \tag{4}$$

where u_1 and u_2 are the control signals applied to the system. They will be determined according to the SMC theory.

SMC is a specific type of variable structure system defined by a decision rule and a number of feedback control laws. The purpose of this control method is to develop the control laws that give an ideal sliding motion on the surface.

The system described in Eq. (1) has only one equilibrium point at zero, so the error dynamics of system (4) are expressed as:

$$\begin{cases} \dot{e}_1 = \alpha(e_2 - e_1 - f(e_1)) + u_1, \\ \dot{e}_2 = e_1 - e_2 + e_3, \\ \dot{e}_3 = -\beta e_2 + u_2. \end{cases} \tag{5}$$

It is seen that $\dot{e}_2 = -e_2$ when e_1 and e_3 are zero in the error dynamics defined by Eq. (5). This means that, the error dynamic e_2 will converge to zero ($e_2 \to 0$) when time goes to infinite ($t \to \infty$). Thus, suitable sliding surfaces can be written as follows:

$$\begin{cases} s_1 = e_1 + k_1 e_2, \\ s_2 = e_3 + k_2 e_2, \end{cases} \tag{6}$$

where k_1 and k_2 must be chosen as positive constant parameters.

The reachability condition for sliding mode is $s\dot{s} < 0$. To fulfill this condition, the SMC signals can be taken as

$$\begin{cases} u_1 = -\alpha(e_2 - e_1 - f(e_1)) - k_1(e_1 - e_2 + e_3) - k_3 s_1 - k_4 \mathrm{sign}(s_1), \\ u_2 = \beta e_2 - k_2(e_1 - e_2 + e_3) - k_3 s_2 - k_4 \mathrm{sign}(s_2), \end{cases} \tag{7}$$

where k_3 and k_4 are constant parameters. Large values of k_3 decrease the time to reach the sliding surface but lead to chattering;