Chaos, Solitons and Fractals 105 (2017) 128–136

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Multiscaling properties on sequences of turbulent plumes images

Pilar López [a,*], Ana M. Tarquis [b], Ania Matulka [c], Benjamin Skadden [b], José M. Redondo [d]

[a] Department of Applied Mathematics (Biomathematics), University Complutense of Madrid, Madrid 28040, Spain
[b] Research and Studies Center for the Management of Agricultural and Environmental Risks (CEIGRAM), University Politécnica of Madrid, Madrid 28040, Spain
[c] Center for Marine Environmental Sciences (MARUM), Department of Geosciences, Geosystem Modeling, Bremen 28359, Germany
[d] Department of Applied Physics, University Politécnica of Cataluña, Barcelona 08034, Spain

ARTICLE INFO

Article history:
Received 13 July 2017
Revised 5 October 2017
Accepted 10 October 2017

Keywords:
Multifractal method
Multifractal spectrum
Turbulent plumes
Entrainment coefficient
Spectrum symmetry

ABSTRACT

A multifractal analysis on a finite-range-scale of the plume concentration images at different experimental conditions (the height of the source H_o), where the measure is the grey value of the image (from 0 to 255), was applied to study its structure through time. The multifractal spectrum showed the characteristic inverse U-shape and a similar evolution in all H_o. The variation of the Hölder exponent ($\Delta\alpha$) presented different amplitudes at different moments and increased with time. The symmetry of the spectrum (Δf) decreased with time achieving negative values (from left hand asymmetry evolving to right asymmetry). We show the different behaviour of axial velocity (W) with $\Delta\alpha$ and Δf. There is a linear relation of entrainment coefficient (α_e) and the entropy dimension (α_1). Therefore, the multifractal spectrum and the derived parameters can be used as markers of plume evolution as well as to study the effect of experimental conditions.

1. Introduction

Turbulent plumes are fluid motions whose primary sources of kinetic energy and momentum flux are body forces derived from density differences [1,2]. The plume boundary is an edge across which the ambient fluid is entrained and the plume boundary moves at the velocity of the plume fluid. In geophysics, it is usually the generation of turbulent plumes as a part of a dispersion process. For example, volcanic plumes or river plumes can be observed where a stream, usually a river, empties into a lake, sea or ocean. Therefore, the geophysical importance of turbulent plumes is clear. It is very interesting to use fractal methods to analyse satellite images to detect and quantify the time behaviour of volcanic plumes to study problems related to environmental impacts or aviation hazards [3,4]. It is also possible to determine the direction of littoral transport at a time if we study the geometry of a river plume. Fractal methods can do this analysis [5]. Larger-scale features of plumes are generally well-represented by the fractal method characterizing scalar isosurfaces in terms of fractal and multifractal properties. Many other processes can be adequately described by fractals, such as river networks [6], rainfall dynamics [7], cloud shapes [8] and turbulent dispersion of a contaminant in the atmospheric boundary layer [9–12].

The fractal technique has been used to analyse turbulent fields in several contexts and provides a natural method for describing the self-similar nature of processes [13]. These are useful tools to analyse the geometric evolution of surfaces in turbulent flows and the implications of this geometric behaviour on mixing [14].

Several studies analyse the relation between the fractal dimension of various surfaces (boundary layers, axisymmetric jets, plane wakes and mixing layers) in high Reynolds number turbulent flows. In 1989, Sreenivasan et al, summarized the previous results on the fractal dimensions of scalar and vorticity interfaces in several classical turbulent flows (a fractal dimension of 2.35 ± 0.05 [15–16]). Fractal dimensions between 1.3 and 1.35 are obtained from LES (large-eddy simulation) plumes for neutral and convective conditions [17]. Prasad and Sreenivasan used the box-counting method to analyse images of jet sections and determined that the fractal dimension of jet boundaries was 1.36, which is close to estimates from atmospheric data [14]. Hentschel and Procaccia predicted a slightly higher cloud perimeter fractal dimension in the range between 1.37 and 1.41 [13]. Sykes and Gabruk determined a fractal dimension (roughly 1.30–1.35) for the scalar concentration field of a turbulent plume dispersion [17]. Lane-Serff investigated the effects of buoyancy on the fractal structure of round, turbulent jets and plumes. He also measured the fractal dimension of concentration contours in jets and plumes, which had an apparent minimum of 1.23 [18].

* Corresponding author
E-mail address: maplopez@bio.ucm.es (P. López).

https://doi.org/10.1016/j.chaos.2017.10.011

Many processes are highly intermittent with spiky measures and strong nonuniformities (for example, the distribution of a turbulent kinetic energy dissipation rate). These intermittent processes cannot be well-described by the typical moment methods and, therefore, a multifractal method is required [19]. Multifractal analysis aims to deduce the multifractal spectrum which is measured for several positive magnitudes that characterize small-scale motions [20–22]. The multifractal description is more general than other theories. Some of these are special cases; for example, the point $f(\alpha = 1) = 1$ in the energy dissipation rate spectrum is Kolmogorov's theory (K41). Actually, we consider that turbulence is a multiplicative process and, therefore, the multifractal method can be used to study turbulence [12]. Puthenveettil et al. conducted the multifractal analysis using binary images and the standard box counting methodology to estimate the multifractal exponents [23]. Finally, Lane-Serff concluded that the use of a single value for the fractal dimension in jets and plumes is questionable [18].

The objective of our study is to apply a multifractal analysis to compare the characteristics of the multifractal spectrum, obtained from grey-scale images of the plume at different times and under different initial conditions, with the classical magnitudes that characterize the plume's dynamic. First, we describe the experimental procedure to generate a turbulent plume, its main characteristics and the multifractal method in Section 2. In §3, we present the multifractal results and their comparison with the axial velocity and the entrainment coefficient of the plume. Finally, in §4 we present the conclusions and we discuss the relevance of this analysis for the presented case study.

2. Material and methods

2.1. Experimental setup and procedures

The aim of the experimental procedure is to generate a turbulent axysimmetric plume, controlling its position and its physical characteristics as buoyancy and momentum fluxes. We release a volume fluid vertically down (with a flow rate up to 8.40 cm^3 s^{-1}) from a small orifice, with a diameter $d = 0.6$ cm, into a stationary body of water with a height of 16.5 cm contained in a glass tank of dimensions 32 cm high and a 25 cm × 25 cm cross-section. The small orifice is located at a height H_o which takes values of 2 cm, 3 cm, 3.5 cm and 6.5 cm and, therefore, increases the overall initial potential energy of the fluid system and the momentum flux. The Atwood number, A, measures the density difference of the fluid system. The Reynolds number at the source, based on the source diameter and the mean velocity there, is approximately 2000. The flow was not observed in the far field (ranging between 250-d and 550-d) due to the dimensions of the tank. The releasing fluid was a potassium permanganate solution (500 cm^3) which is considered as incompressible and miscible and, as such, presents a high Schmidt number (of the order of 10^3) and has an intense purple colour (from pink to mauve). Thus, it was not necessary to add a dye as passive tracer and the flow was directly visualized. A detailed description of the experimental setup can be found in López [24] and in López, Cano and Redondo [25].

The flow was back illuminated from conventional fluorescent lights approximately 0.5 m from the tank giving a projection. This procedure gives an integral image of the plume volume and the registered images average the concentration over the plume volume. The flow was recorded by a high-quality digital video system at high velocity mode (100 fps). The video recordings of the experiments were sequenced into frames using a frame-sequencer software (VirtualDubMod). The frame array had a resolution of 640 × 480 pixels capturing the area of 25 × 18 cm^2. Each frame has intensities recorded as integers in ranging from 0–255. For each experimental video, 288 frames were obtained and between 40 and 60 frames of the time-dependent, three-dimensional plume dispersion were used for the multifractal analysis (those without interaction of the plume with the tank contours).

Fig. 1 shows a sequence of digitized video images from a single experiment and show the time evolution of a turbulent plume. Upon entering the ambient fluid, the source fluid becomes unstable and forms a turbulent plume at the centre of the tank (Fig. 1(a)–(d)). As the plume is gravitationally unstable, it engulfs lighter fluid as it evolves and there is entrainment of the ambient fluid that is directed through the border of the turbulent plume [26]. The downward speed of the plume produces an upward recirculating movement in the ambient fluid which favours the mixing between them.

The behaviour of turbulent plumes is described by three ordinary differential equations for the fluxes of volume, momentum and buoyancy under the Boussinesq assumption [1]. The governing parameters are the radius r, the vertical velocity W, the entrainment velocity U_e and the reduced gravity g'.

The difference between the plume -fluid radial velocity and the total fluid velocity naturally quantifies the purely horizontal entrainment flux of ambient fluid into the plume. This process is characterized by an inflow speed perpendicular to the plume axis which is characterized by the entrainment assumption [1,26–28]. This hypothesis states that the rate of transfer of ambient fluid into the plume, U_E, is proportional to the mean centre-line vertical speed of the plume, W, or axial velocity. The ratio of inflow or entrainment velocity to the plume vertical speed is called the entrainment constant: $\alpha_E = U_E/W$. Fig. 2 shows these main magnitudes that characterize the dynamics of a plume overwritten on the third frame of Fig. 1 ($t = 0.44$ s).

Away from the exit of the nozzle, similarity arguments show that the plume spreads linearly and the axial velocity decreases inversely to the distance. The mean flow model described by Morton et al. gives the plume radius r proportional to the distance from the source z ($r = 6\alpha_P z/5$) where α_p is the entrainment coefficient for a plume and the mean vertical speed W is proportional to r^{-3} ($W \propto r^{-3}$) for plumes [1,18].

Images of plumes, such as other digital imagery, typically contain a large proportion of mixed-pixels (pixels whose digital number is the weighted average of more than one constituent, such as a water/sodium permanganate). To facilitate identification of constituent peaks in the grey-scale histogram, a 2-D filter, executed in NIH ImageJ [29], was run on each frame to mask pixels which differed by more than 0.1% from the surrounding neighbourhood of 25 pixels (5 × 5 unit area). Full details of this technique can be found in Elliot and Heck [30]. The resulting images are shown in Fig. 1(e)–(h).

2.2. Multifractal analysis

Multifractal techniques divide the full image of analysis into boxes to construct samples at different scales. The size of the box for implementing the multifractal method will be the one between the highest resolutions (1 pixel) to the highest size of the full image. A partitioning process starting from the smallest resolution to successively form larger boxes combining pixels is called upscaling process. There are two main methods of upscaling process: the box counting method and gliding box method. In this study, we have applied the latter.

In box counting method the number of boxes will be smaller when the box size ε goes closer to 1. This implies that the number of samples will not be enough for carrying out good statistical analysis, increasing the error associated to the measure. On the contrary, gliding box method construct samples gliding a box over the grid map in all possible ways provided that the box is completely bounded by the grid map. Through this procedure more