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Controlling uncertain Genesio–Tesi chaotic system using adaptive dynamic surface and nonlinear feedback

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ABSTRACT

An adaptive dynamic surface control method using nonlinear feedback is proposed for controlling the uncertain Genesio-Tesi chaotic system. The feature of the nonlinear feedback technique lies in that the feedback gains self-adjust under different amplitudes of system states. Based on the dynamic surface control technique, the complexity explosion problem existing in the backstepping-based chaotic controllers is circumvented. Moreover, the closed-loop stability is guaranteed with rigorous mathematical proof using Lyapunov stability theorem. Comparative results are given to verify the effectiveness and advantage of the proposed method with comparison to the existing linear feedback control.

1. Introduction

Controlling chaos has been a widely studied problem during the last decades [1–4]. During the developments and advances of controlling chaos, several famous chaotic systems have been found and reported, to name a few, the Lorenz system [5], Chua system [6], Chen system [7], Lü system [8], Qi system [9], Genesio system [10], Genesio-Tesi system [11]. As to the methods of controlling chaos, backstepping control using linear feedback has been proved to be an effective methodology [12–19], the closed-loop dynamic can be proved to be stable using the full-fledged Lyapunov stability theorem.

An inherent deficiency of the backstepping design is known as the complexity explosion (CE) problem. Such a problem is caused by the repeated differentiations of the virtual controllers during the recursive design processes [20]. In order to circumvent such a problem, Swaroop et al. proposed the pioneering dynamic surface control (DSC) method [21], where a first-order filter was introduced going after the so-called designed virtual controllers, the output signals were used in the controller design instead of the differentiations of virtual controllers. This method circumvents the repeated differentiations of the virtual controllers, thus, the complexity problem was solved, which has been proved to be a significant success by some subsequent developments and applications [22–26]. The complexity explosion problem also exists in the linear feedback backstepping methods of controlling chaos [12,15,27,28]. These reported methods rely on the high-gain linear feedback to guarantee better controlling performance. However, from the viewpoint of practical applications, high-gain linear feedback control may bring in unexpected high-frequent chattering (HFC) caused by the un-modelled dynamics, it is shown that the stability regions of high-gain closed-loop nonlinear systems may vanish since neglected nonlinearities brings in an unstable limit cycle around an asymptotically stable equilibrium [29].

Among the above-mentioned chaotic systems, the Genesio–Tesi chaotic system [11] generates chaos with less restrictive conditions. Some previous works, for example, [10,17] and the references therein, require the exactly knowing of the system parameters. Some recent advances [13,30] may encounter the above-mentioned CE and HFC problems in practical implementations caused by the direct utilizations of backstepping and linear gain feedback, respectively. Motivated by above observations, this paper studies the problem of controlling Genesio–Tesi chaotic system with uncertain system parameters using a novel nonlinear feedback-based adaptive dynamic surface control, the uncertain system parameters mean that the parameters a, b, and c of condition $ab < c$, which is the key to generates chaotic behaviour of Genesio–Tesi, are not required to be known in the controllers. The major features and contributions of this paper can be summarized as follows:

1. By adding first-order filters after the virtual controllers invoking the DSC technique, the proposed method circumvents the complexity explosion problem in the direct backstepping-based methods in [12–19].
2. With comparison to the linear feedback-based controllers in [12,15,27,28], this paper proposes a *nonlinear feedback-based*

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control using novel continuous differentiable nonlinear feedback function. The major feature of the nonlinear feedback controller lies in that the feedback gains self-adjust with different system states. Speaking in details, the feedback gains decrease with large measured states to avoid the aggressive control actions and potential actuator saturation problems, and increase with small states to attenuation the disturbances and uncertainties.
3. Different from the widely-used quadratic Lyapunov function in [31,32], this paper proposes a non-quadratic Lyapunov function to analyze the closed-loop stability caused by the compound function properties under the nonlinear feedback, and detailed parameter-selection principles are given with rigorous mathematical proof.

The remaining of the paper is organized as follows. Section 2 presents some descriptions of the Genesio–Tesi chaotic system and the formulated problem. In Section 3, adaptive dynamic surface controllers using linear feedback and nonlinear feedback are all given, which facilitates the understanding of the difference between these two methods and the justified comparative results. Section 4 gives the numerical and comparative results to validate the effectiveness and advantages of the proposed method for controlling chaotic Genesio–Tesi system. Section 5 concludes this paper.

2. System description and problem formulation

The dynamics of Genesio–Tesi chaotic system [11] is described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -cx_1 - bx_2 - ax_3 + mx_1^2 + f(X,t) + u(t) \end{cases} \tag{1}$$

where x_1, x_2, and x_3 are the system states, a, b, c, and m are unknown positive real constants satisfying $ab < c$, $X = [x_1, x_2, x_3]^T \in R^3$ is the full-state vector, $f(X, t) \in R$ is the unknown function depending on full-states and time, $u(t) \in R$ is the control signal to be designed. With control input $u(t) = 0$ and $f(X,t) = 0$, the chaotic behavior of Genesio–Tesi system is shown in Fig. 1.

The target is to design the control signal $u(t)$ without requiring the priori knowledge of system parameters such that:

1. The system states track the 3-dimensional target trajectory given as $X_r = [x_{r1}, x_{r2}, x_{r3}]^T = [x_r, \dot{x}_r, \ddot{x}_r]^T$ with arbitrary small error, which is continuous function vector defined on $[t_0, \infty]$. Speaking mathematically, the target is let $X \to X_r$ with $t \to \infty$, such that
$$\lim_{t\to\infty} \|E(t)\| = \lim_{t\to\infty} \|X(t) - X_r(t)\| \to \Omega_0 \tag{2}$$
where Ω_0 is small region around zero, and $\|\cdot\|$ denotes the Euclidean norm of its arguments;
2. The closed-loop system is guaranteed stable in sense that all the closed-loop signals are kept uniformly ultimately bounded.

Without loss of generality, the following assumptions and lemma are needed to design a feasible controller.

Assumption 1. The $f(X, t)$ is bounded with its arguments, that is, there exists some unknown positive constant ϱ, which satisfies $|f(X, t)| \le \varrho, \forall (X, t)$. It is necessary to declare that such positive constant ϱ is not required in the designed controller, which is used only for theoretical analysis.

Assumption 2. The target trajectory x_r and its derivatives $\dot{x}_r$ and $\ddot{x}_r$ are bounded signals.

Lemma 1 (Young's Inequality)**.** *Assume $\mathcal{A}$ and $\mathcal{B}$ are non-negative real number. If $\mathcal{P} > 1$ and $\frac{1}{\mathcal{P}} + \frac{1}{\mathcal{Q}} = 1$, one knows $\mathcal{AB} \le \mathcal{E}\mathcal{A}^{\mathcal{P}} + C_{\mathcal{E}}\mathcal{B}^{\mathcal{Q}}$ with arbitrarily small $\mathcal{B}$ and arbitrarily large $C_{\mathcal{E}}$.*

3. Adaptive dynamic surface control (A-DSC) design

In this section, both the A-DSC designs using linear feedback and nonlinear feedback are given, the former one is partially inspired by existing methods in [12,15,27,28,31], while the latter one improves the linear feedback A-DSC by introducing a new continuous differentiable nonlinear function, which enables the feedback gains self-adjust under difference amplitudes of virtual and actual tracking errors. In the meantime, the differences between the linear and nonlinear feedback based methods are distinct in such manner, the comparative results are thus guaranteed to be quite unprejudiced consequently.

3.1. A-DSC design using linear feedback

In order to proceed the control design, the following error variables are introduced [33]:

$$e_1 = x_1 - x_{r1} \tag{3a}$$

$$e_2 = x_2 - \alpha_1 - x_{r2} \tag{3b}$$

$$e_3 = x_3 - \alpha_2 - x_{r3} \tag{3c}$$

where α_1 and α_2 are virtual controllers of proper arguments to be later designed before the figure-out of actual controller $u(t)$. Similar to the existing literatures, the A-DSC will be worked out using the following three steps.

Step 1: Differentiating both sides of $e_1 = x_1 - x_{r1}$ gives

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{r1} = x_2 - x_{r2} = e_2 + \alpha_1 \tag{4}$$

design the virtual controller α_1 as follows:

$$\alpha_1 = -k_1 e_1 \tag{5}$$

where k_1 is the positive design parameter. To avoid the differentiation of α_1 in the following design steps, let α_1 will pass through a first-order filter to generate a new variable φ_1:

$$\tau_1 \dot{\varphi}_1 + \varphi_1 = \alpha_1, \;\; \varphi_1(0) = \alpha_1(0) \tag{6}$$

where τ_1 is the time coefficient of filter, and $\varphi_1(0)$ and $\alpha_1(0)$ are the initial values of φ_1 and α_1, respectively.

Remark 1. The virtual controller α_1 is designed as the direct constant negative feedback of e_1. Incorporating (5) with (4), the virtual closed-loop system can be obtained as $\dot{e}_1 = -k_1 e_1 + e_2$. In this situation, one chooses the Lyapunov function $V_1 = \frac{1}{2} e_1^2$ and its time derivative is $\dot{V}_1 = e_1 \dot{e}_1 = -k_1 e_1^2 + e_1 e_2$. Because the e_2 is unknown information in this step, we need now proceed to the following step to figure out such an issue. And the introduction of new variable φ_1 in (6) is to circumvent the repeated differentiations of α_1 in the following steps, this is inspired by the pioneering DSC method proposed in [21]. Similar arguments also apply to the following design steps.

Step 2: Differentiating $e_2 = x_2 - \alpha_1 - x_{r2}$ gives

$$\dot{e}_2 = x_3 - \dot{\alpha}_1 - \dot{x}_{r2} = e_3 + \alpha_2 - \dot{\alpha}_1 \tag{7}$$

design the virtual controller α_2 as follows:

$$\alpha_2 = -k_2 e_2 + \dot{\varphi}_1 \tag{8}$$

where k_2 is the positive design parameter. Similar to Step 1, variable φ_2 are introduced by the following first-order filter:

$$\tau_2 \dot{\varphi}_2 + \varphi_2 = \alpha_2, \;\; \varphi_2(0) = \alpha_2(0) \tag{9}$$

where τ_2 is the time coefficient of filter, and $\varphi_2(0)$ and $\alpha_2(0)$ are the initial values of φ_2 and α_2, respectively.