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Generation of a family of fractional order hyper-chaotic multi-scroll attractors

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A B S T R A C T

An unified method to yield a family of fractional-order (FO) hyper-chaotic multi-scroll (HCMS) systems in R^n is proposed. Firstly, a new simple 3-dimensional (3-D) FO unstable linear system is introduced. Afterwards, additional variables are added and one nonlinear controller with adjustable parameters is included to generate HCMS attractors. A guideline to construct HCMS systems of any dimension is presented, that is verified along within the dynamics of three examples, namely 4-D, 5-D and 10-D FO HCMS systems. Phase portraits, Poincaré maps and two positive Lyapunov exponents are calculated. Moreover, a circuit of 0.96-order is also designed to realize one 4-D FO HCMS system. Numerical simulations and circuit simulation results show the feasibility of the novel approach.

1. Introduction

Chaos has been investigated widely in the last decades and become a subject of increasing interest because of its great potential in many fields such as digital data encryption, secure communication, mechanics and other applications [1,2]. In 1963, Lorenz found the first chaotic attractor in a 3-D autonomous system. Later, Chua invented the famous double-scroll Chua circuit, which brought the research of chaos into a new era. The Chua double-scroll circuit is the first physical implementation of chaos, which has been intensively investigated after its discovery [3]. Since then, other systems with complex chaotic attractors, such as multi-directional multi-scroll attractors [4–7], hyper-chaotic attractors [8–10], and hyper-chaotic multi-scroll attractors [11–13] have been constructed.

Hyper-chaos was discovered by Rössler in [14]. The most significant difference between chaos and hyper-chaos is that hyper-chaotic systems have more than a single positive Lyaponov exponent (LE) and show complex dynamic effects. For example, three coupled rods with a horizontally situated barrier exhibits very rich nonlinear dynamics with 2 positive LEs[15], motion of the pendulum shows chaotic attractor with 3 positive LEs [16], analysis of the hyper, hyper-hyper and spatial-temporal chaos of continuous mechanical systems with 2, 3 and 4 positive LEs were considered in [17,18]. In fact, it was recognized that hyper-chaotic systems are more unpredictable and random-like than standard chaotic systems. Therefore, hyperchaos is preferred in many applications including secure communications, chaos-based image encryption and cryptography, among others. Recently, research showed that chaos exists not only in integer-order systems, but also in FO systems. It has been reported that several systems described by FO models display complex bifurcation and chaos phenomena. For example, FO Duffing oscillator [19], and systems such as FO Rössler [20], FO Chen [21], FO Lorenz [22], FO Lü [23], and FO Liu [24] models reveal complex dynamics. Research about FO systems mainly focused on two aspects: (i) to construct new 4-D hyper-chaotic systems by adding additional state variables to a 3-D FO chaotic system [25–28], and (ii) to generate FO multi-scroll attractors by employing different kinds of function series [29–35]. Naturally, this raises two interesting questions. How can we create FO HCMS attractors artificially? Can we generate HCMS attractors of any pre-defined dimension by adding controllers to an unstable FO system? To the best of our knowledge, little attention has been paid to these problems.

Inspired by the above discussion, a family of n-dimensional FO HCMS systems is constructed from an unstable low dimensional FO linear system. For this purpose, two steps need to be executed. First, extra variables must be added to allow the system to

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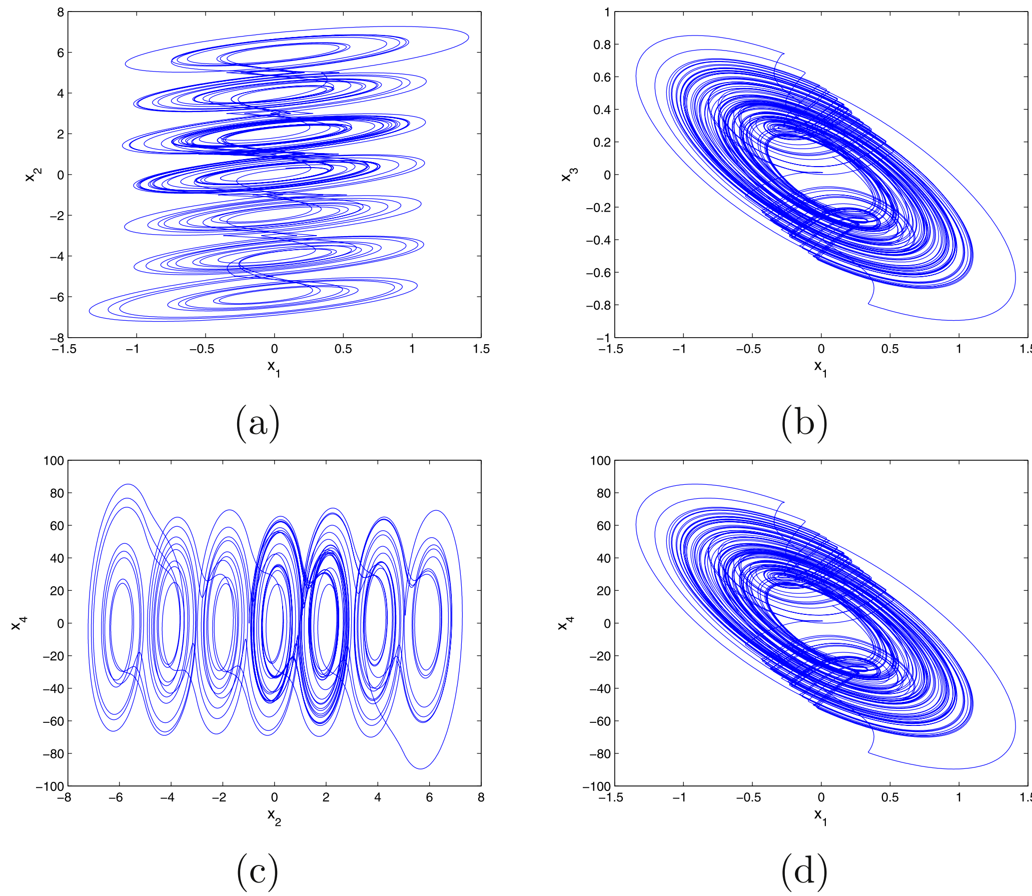

Fig. 1. The phase portrait of example 1 in Table 1: (a) $x_1 - x_2$; (b) $x_1 - x_3$; (c) $x_2 - x_4$;(d) $x_1 - x_4$.

Table 1
Different portraits with different parameters.

Example	Parameters						Portrait
	a_1	a_2	k	μ	N_1	N_2	
1	100	0	0	2	3	0	Fig. 1
2	1	1	1	2	3	3	Fig. 2
3	5	2	1	2	3	3	Fig. 3
4	1	1	1	2	3	1	Fig. 4

be hyper-chaotic. Second, multi-scroll attractors need to be constructed. In fact, it is straightforward to accomplish the second step, but the crucial question is the first, that is, how to guarantee that we will obtain a hyper-chaotic system? In this paper, the processes of adding extra variables and designing a feasible controller to solve the problem will be analyzed. In this perspective, phase portraits and Poincaré maps of the systems are depicted, and two positive Lyapunov exponents are calculated. Moreover, a simulation circuit with 0.96-order is implemented, and multi-scroll attractors are observed.

The paper is organized as follows. In Section 2 the preliminaries of fractional calculus are introduced and a new initial model is described. In Section 3 the 4-D and 5-D FO HCMS systems are constructed, and the generation mechanism of multi-scroll attractors is proposed and verified by means of the 5-D FO HCMS system. In Section 4 a circuit is designed to realize the 4-D FO HCMS system. Finally, in Section 5 the conclusions are drawn.

2. Preliminaries and model description

There are three commonly used definitions of FO derivatives, namely the Caputo, the Riemann–Liouville and the Grünwald–Letnikov formulations. In this paper, we adopt the Caputo derivative because of its convenience in applied sciences.

Definition 1. [36] The FO integral (Riemann–Liouville integral) $D_{t_0,t}^{-\alpha}$ of order $\alpha \in R^+$ of function $x(t)$ is defined as:

$$D_{t_0,t}^{-\alpha}x(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}x(\tau)d\tau,$$

where $\Gamma(\cdot)$ is the gamma function, $\Gamma(\tau) = \int_0^\infty t^{\tau-1}e^{-t}dt$.

Definition 2. [36] The Caputo FO derivative of order α of function $x(t)$ is defined as:

$$D^\alpha x(t) = D_{t_0,t}^{-(n-\alpha)}\frac{d^n}{dt^n}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{(n-\alpha-1)}x^{(n)}(\tau)d\tau,$$

where $n-1 < \alpha < n \in Z^+$.

Definition 3. [32] Considering a general n-dimensional FO system:

$$D^\alpha(X) = AX + B(x), \tag{1}$$

the roots of the equation $AX + B(x) = 0$ are called the equilibrium points of the FO differential system, where $D^\alpha(X) = (D^\alpha x_1, D^\alpha x_2, \ldots, D^\alpha x_n)^T$, $X = (x_1, x_2, \ldots, x_n)^T \in R^n$. Here, A is the coefficient matrix, B is the nonlinear controller, and α is the FO.

Definition 4. [37] For an n-dimensional FO system, suppose that there is one saddle point O, and that λ_i are the eigenvalues of the Jacobi matrix J at O. If $\sum_{i=1}^n \lambda_i < 0$, J has at least one real negative eigenvalue and at least one pair of conjugate complex eigenvalues $\delta \pm j\omega$, $\delta > 0$, $\omega > 0$, $\arctan(\omega/\delta) < \alpha\pi/2$. If α is the FO of the system, then O is called an equilibrium point with index 2. For each