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Conditional punishment resolves social dilemma in spatial network

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ABSTRACT

Social punishment, a mechanism that cooperative individual spends a little cost to penalize defector, is verified to be an effective mechanism for promoting the evolution of cooperation. In this paper, we introduce conditional punishment, the willingness to punish p, which decides whether to carry out penalty. It is shown that cooperative behavior is significantly enhanced when punishers are taken into account and the frequency of cooperation increases with p. In addition, we find out the protective effect of punishers on evolution of cooperation from a micro point of view. We hope our work may shed light on understanding of cooperative behavior in society.

1. Introduction

Cooperation is an ubiquitous phenomenon in the real world, ranging from ecosystems to social systems [1,2]. In recent years, the emergence and maintenance of cooperation have attracted considerable attention from multiple disciplines and have made great breakthroughs and progress. However, there are still some questions that remain to be solved [3,4].

In this case, a classical mathematical framework named evolutionary game theory, which is widely used, has provided a powerful tool to explore the evolution of cooperative behavior [5–8]. Furthermore, the prisoner's dilemma (PD) game, as a typical metaphor, has also been investigated through pairwise interaction extensively [9,10]. In the PD, two players make a choice that determines payoffs between cooperation (C) and defection (D). They will receive the reward R if both of them choose to cooperate and the punishment P for mutual defection. If a cooperator encounters a defector, the former will get the sucker's payoff S while the latter will obtain the temptation to defect T. These payoffs satisfy the order $T > R > P > S$ and $2R > T + S$, which means that defection is the best choice regardless of the opponent's strategy and will give rise to the extinction of cooperation eventually. However, mutual cooperation will lead to more total benefits than mutual defection.

In order to solve this dilemma, a lot of novel mechanisms have been proposed [11–21]. The seminal works by Nowak and May found that cooperators could prevail via forming compact clusters on complex networks, which is regarded as network reciprocity on account of spatial structure [22,23]. Moreover, the theory of network reciprocity motivates plenty of interesting study to resolve the dilemma. For instance, the influence of environment [24,25], asymmetric [26], reputation [27], aspiration [28], the heterogeneity of the spatial structure and asynchronous update allow the cooperation to prevail even when temptation is pretty high. In addition, the study has also expanded to different topologies, such as small-world network, scale-free network, random regular network, multilayer network and so on [29,30].

Among above mechanisms, what attracts our interests most is social punishment, which has received relatively little attention in PD until now. Social punishment is a behavior that punishes free riders by cooperators, which is a common phenomenon in the real world. During this process, cooperators will spend a little cost to punish defectors by a fine. Most of this work have already investigated in public goods (PG) games, where each member of a group starts with an endowment and decides how much to contribute to the public pool [31–33]. Under this setup, punishment in PG games is divided into peer punishment and pool punishment. Nevertheless, some studies introduce social punishment into PD game recent years [34,35]. For example, Wang et al. found that social punishment can promote cooperation obviously in PD and snowdrift (SD) game no matter with which network structure. However, punishers have to pay for a small cost themselves, so someone wouldn't like to punish the free riders. Based on this, we introduce the willingness to punish p ($0 \le p \le 1$) to control the proportion of punishers and investigate its influence.

In this paper, we will introduce a probability to command the social punishment and explore the evolution of cooperation under the setting that punishment is implemented with a fixed probabil-

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Table 1
Payoff matrix of prisoner's dilemma with social punishment. C, D and P represent cooperation, defection and punishment respectively. Here, γ stands for the cost of punishment and β is the fine assumed by defectors.

	C	D	P
C	R	S	R
D	T	P	T-β
P	R	S-γ	R

ity. In the rest of this paper, we will first describe the evolutionary PD games with social punishment and the details of our mechanism in the next section. Section 3 is devoted to the presentation of results, whereas the main conclusion will be given in the last section.

2. Model

We introduce the willingness to punish into prisoner's dilemma (PD) game and consider evolutionary games on a regular $L \times L$ square lattice with periodic boundary condition. Each player is randomly designed as cooperator (C), defector (D) or punisher (P) with equal probability initially. That is to say, each strategy covers one-third of the square lattice. Without loss of generality, we set $R = 1$, $T = b$ and $P = S = 0$, where $1 < b \leq 2$ ensures a proper payoff ranking ($T > R > P \geq S$) of a simply and weak version. In particular, cooperators that punish defectors are introduced as the third competing strategy, that is, punishers (P). Punishers will act as cooperators if they encounter cooperators and themselves. However, punishers will cost γ to punish defectors by a fine β additionally with a probability p when in the interaction with defectors. The payoff matrix is displayed in Table 1. Otherwise, punishers will always play the role of cooperators.

The standard Monte–Carlo (MC) simulation procedure comprises the following elementary steps. First, focal player x obtains his payoff P_x that calculated in accordance with above-mentioned by interacting with his four nearest neighbors at each time step,

$$P_x = \sum\nolimits_i^n P_i, \tag{1}$$

where $n = 4$ denotes the interaction neighborhood size of player x. Then one randomly chosen neighbor y also acquires his payoff P_y in the same way. If $P_x > P_y$, player x will remain his strategy in the next time step. On the contrary, player x will adopt the strategy s_y of player y with a probability proportional to the maximum payoff difference while $P_x < P_y$:

$$W(s_x \leftarrow s_y) = \frac{P_y - P_x}{n \times \Delta}, \tag{2}$$

where Δ represents the maximum possible payoff difference between two players.

During one full MC step, each player has a chance on average to update his strategy and change the game he played as described above. The simulation result is carried out on 100 × 100 lattices with periodic boundary. The cooperator density ρ_C was determined by averaging the last 5×10^3 steps over total 5×10^4 MC steps. Furthermore, final data results from averaging over 20 realizations.

3. Results

First, we check the relationship between the fraction of cooperation ρ_c and the temptation to defect b for different value parameter p and given $\beta = 0.5$, $\gamma = 0.1$ in Fig. 1. When p equal to zero, it returns back to traditional model that there is no punisher exist in the population. So, the fraction of cooperators down quickly with the increase of b, even diminish when b is small. However, when we consider the willingness to punish p, cooperators can survive from defectors' invasion for large b. With the increase of parameter p, cooperative behavior is promoted effectively. When $p = 1$, all punishers will punish the selfish individual, thus cooperators can keep a high level. Furthermore, large p means a higher punish willingness, and these results totally suggest that large proportion punishers can promote cooperation. Especially, the larger the value of p, the higher is the level of cooperation.

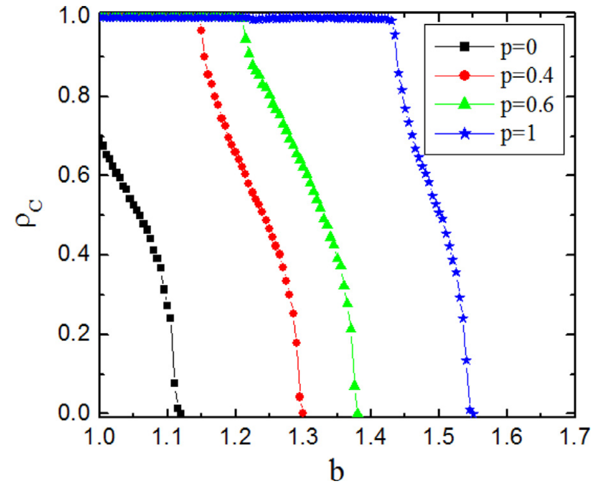

Fig. 1. The density of cooperators (ρ_c) as function of b for different p. When $p = 0$ (traditional version), cooperators will diminish even b is small. With the increase of parameter p, the cooperators can resist the invasion of defectors effectively through the influence of punishers. All the results are obtained for $\beta = 0.5$, $\gamma = 0.1$.

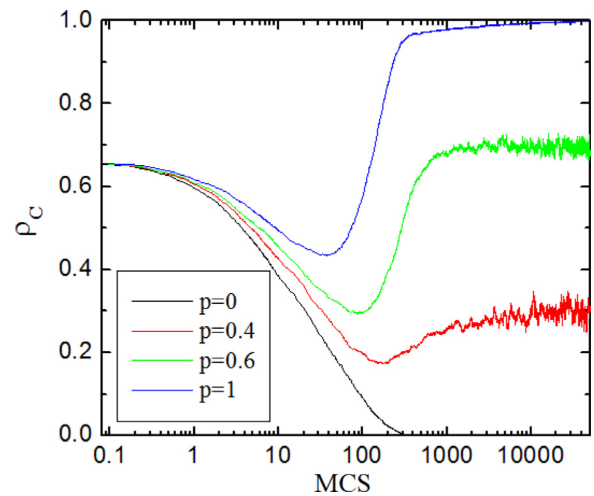

Fig. 2. Time evolution of the cooperative behavior (ρ_c) on square lattices for $p = 0$, 0.4, 0.6, 1 from bottom to up. With the increase of p, the frequency of cooperators increases obviously at stable state and it even dominant the lattice when $p = 1$, defectors have no chance to survive. All the results are obtained for $b = 1.28$, $\beta = 0.5$, $\gamma = 0.1$.

In order to investigate the influence of punish willingness on the evolution of cooperation, we show the time course of ρ_c for given $b = 1.28$, $\beta = 0.5$ and $\gamma = 0.1$ with different p in Fig. 2. Initially, cooperators, defectors and punishers are randomly distributed on square lattice. When $p = 0$ (traditional version), the density of cooperators down rapidly and go extinct with the increase of time steps, because defectors is more successful than cooperators. When $p > 0$, the density of cooperators will down to a