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Vital layer nodes of multiplex networks for immunization and attack

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ABSTRACT

When dealing with the optimal prevention of epidemics or destruction of network structures, one important question that can be asked is the location of vital nodes which need to be immunized or removed first. In the last decade, the vital nodes identification has attracted increasing attentions. However, the majority of the existing achievements are limited to single networks, how to identify the vital nodes of multiplex networks need further exploration. The nodes of multiplex networks can be divided into two categories: multiplex node (MN) and layer node (LN). In this paper, we focus on identifying the vital LNs of multiplex networks for immunization or attack. We extend several indexes or algorithms from single networks to multiplex networks, including high degree, high betweeness and their variations based on adaptive strategies, and the collective influence, explosive immunization and simulated annealing, to identify the vital LNs. By performing them on different kinds of multiplex networks, we find the explosive immunization is always the best for the identification of vital LNs. Particularly, the performances of the proposed indexes and algorithms could be improved considerably when the greedy reinserting strategy is considered except the explosive immunization, which however still performs the best. Our work offers a deeper understanding for the vital nodes identification in multiplex network and provides novel insights for further studies of the immunization and attack on multiplex networks.

1. Introduction

Over the past years, complex networks have proven to be a successful tool in describing a large variety of real-world complex systems, ranging from biological, technological, social infrastructures to information, engineering, and physical systems [1–3]. It was found that most of the real complex networks display power law shaped degree distribution $P(k) \sim k^{-\gamma}$, with exponents varying in the range $2 < \gamma < 3$ [4]. That is, the most of real networks, having a highly heterogeneous degree distribution, result in the simultaneous presence of a few nodes linked to many other nodes, and a large number of poorly connected elements. Naturally, the heterogeneous or the so called scale-free property makes the roles of different nodes different in the structure and function of the network. For example, immunizing some of vital nodes (high degree or high betweeness, etc.) could provide a more efficient way to fragment the population network and thus cut the propagation routes and reduce the possibility of large disease outbreaks, than immunizing same number of random chosen nodes [5,6]. In addition, removing vital nodes could destroy the network connectivity and break the network functionality more efficiently, than just attacking a random chosen nodes set with the same size [7,8].

Identifying the vital nodes of networks has a lot of potential applications [9], typically including better control the outbreak of epidemics, conduct successful advertisements for e-commercial products, prevent catastrophic outages in power grids or the Internet, optimize the use of limited resources to facilitate information propagation, discover drug target candidates and essential proteins, maintain the connectivity or design strategies for connectivity breakdowns in communication networks, identify the best player from the records of professional sport competitions, and predict successful scientists as well as popular scientific publications based on co-authorship and citation networks [10–19]. However, identifying the vital nodes of networks is not a trivial task. As a result, existing works instead propose heuristics to approximately solve it, most of which are based on structural centrality measures of networks, including degree, betweenness, pagerank, closeness, eigenvector, etc. Moreover, the centrality can be improved by adaptive strategies in which the centrality is recalculated for the network of unimmunized or unremoved nodes at each step [9].

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Recently, a few of excellent methods or algorithms on the identification of vital nodes were proposed and has attracted a lot of enthusiasm in the network science community. By mapping the identification of vital nodes onto optimal percolation, Morone and Makse present the Collective Influence (CI) algorithm which is able to produce nearly optimal solutions in almost linear time, and performs better than any other methods [16]. After that, Zhou et al. built connection between the network dismantling and the network decycling (the removal of nodes leaving the network acyclic), and propose belief propagationguided decimation (BPD) algorithm to solve the network dismantling problem, which equivalent to find the vital nodes of networks for immunization or attack. BPD has better performance than CI and almost of the other centrality indexes [17]. At about the same time, an independent work was down by Braunstein et al. who propose a novel decycling algorithm based on the message passing to the network decycling problem [18]. In addition, Clusella et al. consider the site percolation transition on network on which all links are already present, but nodes are still to be inserted one by one. They propose the algorithm called explosive immunization, which proceeds by first inserting the low score nodes that are not immunized or removed [19].

As above described, though there have been many achievements, most of them are mainly concentrated to the case of single network which treats all the networks links on an equivalent footing. In the past few years, we have witnessed a tremendous shift that the focus of complex networks was extended from single network to multiplex network which is composed of several network layers constructed by same nodes but with different topologies and dynamics [20–28]. Multiplex networks could explicitly capture the authentic and natural characteristics of real world systems: the same node may have different kinds of interactions and each channel of connectivity is represented by a layer. Zhao et al. divide the nodes of multiplex networks into two categories: multiplex node (MN) and layer node (LN, the agent or replica of MN in each layer) [29]. Identifying the vital nodes of multiplex networks naturally becomes a very interesting and crucial challenge. Osat et al. reframe several algorithms, including degree, collective influence, explosive immunization and simulated annealing for vital MNs identification from single networks to multiplex networks [30]. S Ribalta, M Romance and Halu, et.al. proposed the centrality measures of MNs like betweenness, rigenvector, pagerank and clustering coefficients etc [31–35]. However, how to identify the vital LNs of multiplex networks is still virgin and needs further exploration [36–38].

In this paper, we focus on identifying the vital LNs of multiplex networks such that, if these nodes are removed, the multiplex network is maximum fragmented into many disconnected clusters of non-extensive size. We extend several indexes or algorithms from single networks to multiplex networks, including high degree, high betweeness and their variations based on adaptive strategies, and the collective influence, explosive immunization and simulated annealing, to identify the vital LNs. We also test and compare the performances of these algorithms on different kinds of multiplex networks.

2. Network models and problem definition

Multiplex network is the combination of several single network layers which contain the same nodes or the agent or replica of same nodes yet different intra-layer connections. They are genuine representations for many real-world systems, including social networks, technological networks, transportation networks, etc. Fig. 1 gives an illustration of a multiplex framework: five people are connected via two types of social relationships, the Twitter connections (blue links) and the Facebook connections (black links) (see top panel). The node in this network is denoted by multiplex node (MN). Such systems can be well embedded into the framework of multiplex networks with two types of links. The link type in the system defines a network layer, and the nodes of each network layer are, called layer nodes (LN), the agents or replicas of the MNs in that layer (see bottom panel). A similar example is shown in Fig. 2 where five cities are connected by airway and railway. Therefore, the cities can be as the MNs of the multiplex city networks, and the agents (the airport and railway station) of the MN in each layers can be regarded as the LNs.

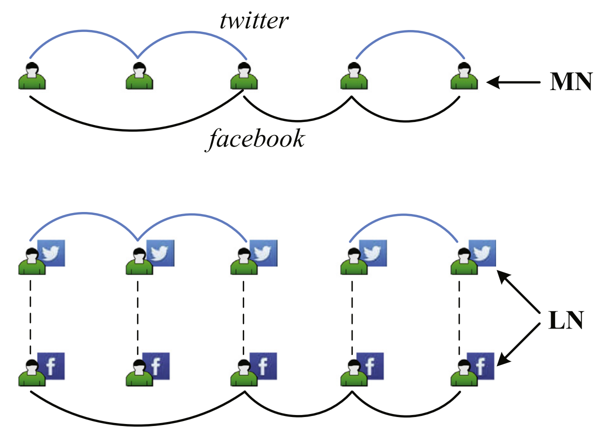

Fig. 1. An illustration of multiplex framework and the representations of MNs and LNs.

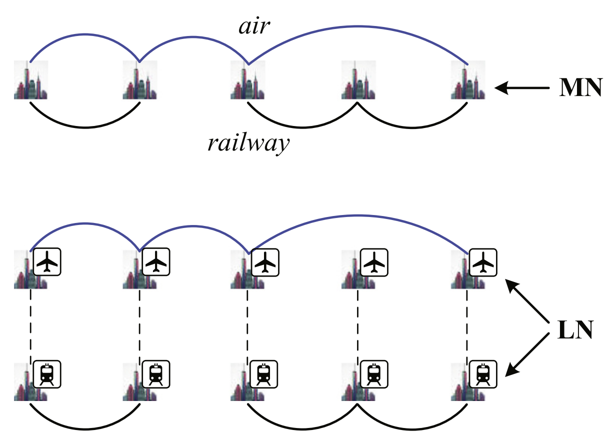

Fig. 2. An illustration of multiplex framework and the representations of MNs and LNs.

When consider the identification of vital nodes of networks, the natural measure we often used is the size of the giant connected component (GCC) as the vital nodes are immunized or removed from the network. *It is noted that the nodes of the GCC in multiplex networks should be the MNs, while the function of MN depends on its LNs. That is the size of the GCC of multiplex networks represents the number of MNs which belong to the GCC, and an MN belongs to the GCC as long as it could connect to the GCC through at least one of its LNs.* Fig. 3 gives an example that some of the LNs (social media accounts) of the multiplex social networks are immunized (forbidden), the MNs (people) still can communicate with others through their unimmunized LNs. Specifically, when LNs, 1^t, 5^t, 2^f and 5^f are removed from the multiplex networks, the MNs, 1, 2, 3 and 4 belong to the GCC of the resulting networks since they can connect to the GCC via at least one of their LNs.

In this paper, we focus on identifying the vital LNs of multiplex networks $G = \{1, 2, \ldots, N\}$ coupled by two network layers $G^\alpha = \{1^\alpha, 2^\alpha, \ldots, N^\alpha\}$ and $G^\beta = \{1^\beta, 2^\beta, \ldots, N^\beta\}$. i^α and i^β are the