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Synchronization in a network of delay coupled maps with stochastically switching topologies

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ABSTRACT

The synchronization behavior of delay coupled chaotic smooth unimodal maps over a ring network with stochastic switching of links at every time step is reported in this paper. It is observed that spatiotemporal synchronization never appears for nearest neighbor connections; however, stochastic switching of connections with homogeneous delay (τ) is capable of synchronizing the network to homogeneous steady state or periodic orbit or synchronized chaotically oscillating state depending on the delay parameter, stochasticity parameter and map parameters. Most interestingly, linear stability analysis of the synchronized state is done analytically for unit delay and the value of the critical coupling strength, at which the synchronization occurs is determined analytically. The logistic map $rx(1-x)$ (a smooth unimodal map) is chosen for numerical simulation purpose. It is found that synchronized steady state or synchronized period-2 orbit is stabilized for delay $\tau = 1$ depending upon the nature of the local map. On the other hand for delay $\tau = 2$ the network is stabilized to the fixed point of the local map. Numerical simulation results are in good agreement with the analytically obtained linear stability analysis results. Another interesting observation is the existence of synchronized chaos in the network for delay $\tau > 2$. Calculating synchronization error and plotting time series data and Poincare first return map and largest Lyapunov exponent the existence of synchronized chaos is confirmed. The results hold good for other smooth unimodal maps also.

1. Introduction

There has been an increasing interest in the study of spatially extended systems with local and nonlocal interactions to understand almost all complex behaviors in nature. The analysis of collective behavior of time-delay coupled networks is a topic of great interest for its fundamental significance from a dynamical systems point of view and for its practical relevance in modeling various physical, biological, and engineering systems, such as coupled laser arrays, gene regulatory networks and complex ecosystems [1–3]. Realistic modeling of many large networks with nonlocal interaction inevitably requires connection delays to be taken into account, since they naturally arise as a consequence of finite information transmission and processing speeds among the units. Delay systems are in general infinite dimensional and can display complex dynamics and a very little is known yet about the basic relations between network structure and delay dynamics. The first systematic investigation of delay coupled phase oscillators was done by Schuster and Wagner [4]. Delay can give rise to some significant phenomena like synchronization [5–7], multistability [8,9] and amplitude death [10,11]. Dissipative systems with time delayed feedback can generate high dimensional chaos and the dimension of chaotic attractor can be made larger by increasing delay time [12–15]. In neural network and communication network time delays ubiquitously exist [16,17]. Networks with time delay are also very useful for modeling coordinated brain activity [18], pattern recognitions etc. Delay can enhance the coherence of chaotic motion [19], as well as it can stabilize synchronous state in oscillator network [20]. Some new phenomena like oscillation death, stabilizing periodic orbits, enhancement or suppression of synchronization, chimera state etc arise due to the time delay [21–26]. So there are strong reasons to shed light on the issue of such systems.

Synchronization of coupled map lattice under delay coupling in globally coupled logistic maps [27,28], coupled chaotic maps with inter-neural communication [29] or intra-neural communication [30] has been intensively investigated. In modeling many realistic systems of biological, technological and physical significance using CML with either nearest neighbor interaction or global coupling is unable to capture all essential features of the dynamics of the extended systems [31]. However, in many systems (for

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example, communication, ecological, social and contact networks) links are not always active and the connectivity between the units changes (stochastically or deterministically) in time with a rate ranging from slow to fast [32]. One can think that time variations of links represent the evolution of interactions over time of the system. Such time-varying interactions are commonly found in social networks, communication, biological systems, spread of epidemics, computer networks, world wide web etc. and have been shown to result in significantly different emergent phenomena [33,34].

The synchronization behavior of time delay coupled network with dynamic random updating of links at every time are not yet investigated. In this work, we have considered homogeneous delay coupled ring network with stochastic updating of links at every time and investigated the effects of dynamic random updating of links and delay parameter on the synchronization phenomena of the network. Most interestingly, we study the stability of the synchronized state when the underlying network evolves in time taking delay parameter as one unit of time. We analyze synchronization behavior of dynamic random network with identical local maps. We consider Watts-Strogatz (WS) networks and vary the fraction of random links p to cover a broad range of networks varying from a regular ring topology for $p = 0$ to completely random networks for $p = 1$ and for intermediate values of p, such networks are characterized by small-world networks : small path length and high clustering coefficient.

The paper is organized as follows: In Section 2 the model is discussed. Section 3 contains linear stability analysis results for complete synchronization. Numerical simulation results are also presented and analyzed in this section. Finally, conclusion is drawn in Section 4.

2. Model

In our model the governing equations for delay coupled maps with nearest neighbor interaction over a ring network are the following

$$x_{t+1}(i) = (1-\epsilon)f(x_t(i)) + \frac{\epsilon}{2}(x_{t-\tau}(i-1) + x_{t-\tau}(i+1)) \tag{1}$$

where, $x_t(i)$ represents the state variable, $t(\geq 0)$ is the integer valued time index, $i = 1, \ldots N$ are the space index, N is the linear size of the array. ϵ is the coupling strength, $\tau \geq 0$ represents the constant delay time. In our model every link in the network rewires stochastically and independently in every time step. In particular, at every time update we connect a fraction p of randomly chosen sites in the lattice to two other random sites, instead of their nearest neighbors. Then the evolution rule for p fraction of nodes of the network are followed by

$$x_{t+1}(i) = (1-\epsilon)f(x_t(i)) + \frac{\epsilon}{2}(x_{t-\tau}(\xi) + x_{t-\tau}(\eta)) \tag{2}$$

where ξ and η are random integers uniformly distributed in the set {1, 2, 3, ..., N}. At the same time the dynamical equation of the time evolution rule for remaining $(1-p)$ fraction of nodes are governed by

$$x_{t+1}(i) = (1-\epsilon)f(x_t(i)) + \frac{\epsilon}{2}(x_{t-\tau}(i-1) + x_{t-\tau}(i+1)) \tag{3}$$

Thus $p = 0$ corresponds to the usual nearest-neighbor diffusion for all nodes, whereas $p = 1$ is related to completely random diffusion. For our present study we choose one dimensional unimodal smooth map.

3. Results

In this section, we shall do linear stability analysis of synchronized spatiotemporal fixed point of the dynamic random network of delay coupled maps first.

3.1. Stability analysis

We consider the case with delay parameter $\tau = 1$ and check the stability of the synchronized spatiotemporal fixed point of the map. Stability analysis of synchronized spatiotemporal fixed point for dynamic random network is relatively complicated since it will involve diagonalization of random matrices. To calculate the stability of the synchronized fixed point, we will construct an average probabilistic evolution rule for the sites, which becomes a sort of mean field version of the dynamics [35]. We assume that the total contribution due to k randomly chosen neighbors is $k <x(t)>$. Now, the averaged out evolution equation for any site i is the following

$$x_{t+1}(i) = (1-\epsilon)f(x_t(i)) + (1-p)\frac{\epsilon}{2}(x_{t-\tau}(i+1) + x_{t-\tau}(i-1)) + \frac{p\epsilon}{N}\sum_{j=1}^{N} x_{t-\tau}(j). \tag{4}$$

The delayed system (4) can now be written in following form by introducing a new variable $y_t(i)$ as

$$\begin{aligned} x_{t+1}(i) &= (1-\epsilon)f(x_t(i)) + (1-p)\frac{\epsilon}{2}\{y_t(i+1) + y_t(i-1)\} + \frac{p\epsilon}{N}\sum_{j=1}^{N} y_t(j) \\ y_{t+1}(i) &= x_t(i) \end{aligned} \tag{5}$$

Now it becomes an equation with no delay term. In this way every coupled map lattice model with homogeneous delay or finite number of deterministic delays can be easily transformed into an CML with no delay. Synchronization occurs when the state variables of each node adopt the same value for all the coupled maps at all times t, i.e., $x_t(1) = x_t(2) = x_t(3) = ... = x_t(N) = x^*$ and $y_t(1) = y_t(2) = y_t(3) = ... = y_t(N)$ for all times greater than a transient time. A range of the coupling strength may exist for which such a synchronized steady state can be obtained depending on the nature of the individual map and an adequate rewiring probability p and delay parameter τ. The linear stability analysis of synchronized spatiotemporal steady state of the model (5) is performed for smooth unimodal maps. The Jacobian matrix for the system (5) can be cast as

$$J = \begin{bmatrix} A+C & B+C & C & C & \ldots & B+C \\ B+C & A+C & B+C & C & \ldots & C \\ \vdots & \vdots & \vdots & \vdots & \ldots & \vdots \\ B+C & C & C & C & \ldots & A+C \end{bmatrix} \tag{6}$$

where,
$A = \begin{pmatrix} (1-\epsilon)a & 0 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & (1-p)\epsilon/2 \\ 0 & 0 \end{pmatrix}$ and
$C = \begin{pmatrix} 0 & p\epsilon/N \\ 0 & 0 \end{pmatrix}$, $a = \frac{df(x)}{dx}|_{x=x^*}$.

To derive the stability condition of the synchronized steady state we will study the eigenvalues of the matrix J. The matrix J is a block circulant matrix. It can be reduced to a block diagonal matrix using unitary transformation. The block diagonal form of J is the following

$$D = \begin{pmatrix} M_0 & 0 & \ldots & 0 \\ 0 & M_1 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & M_{N-1} \end{pmatrix} \tag{7}$$

where, the matrix $M_r (r = 0, 1, 2, \ldots N-1)$ are 2×2 matrices given by