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Bifurcations and chaos of the nonlinear viscoelastic plates subjected to subsonic flow and external loads[☆]

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ABSTRACT

The subharmonic bifurcations and chaotic motions of the nonlinear viscoelastic plates subjected to subsonic flow and external loads are studied by means of Melnikov method. The critical conditions for the occurrence of chaotic motions are obtained. The chaotic features on the system parameters are discussed in detail. The conditions for subharmonic bifurcations are also obtained. For the system with no structural damping, chaotic motions can occur through infinite subharmonic bifurcations of odd orders. Furthermore, we confirm our theoretical predictions by numerical simulations. The theoretical results obtained here can help us to eliminate or suppress large nonlinear vibrations and chaotic motions of the nonlinear viscoelastic plates. Based on Melnikov method, complex dynamical behaviors of the nonlinear viscoelastic plates can be controlled by modifying the system parameters.

1. Introduction

Composite plates are widely used in many engineering fields such as automobiles, aircrafts, robot arms and submarines. Consequently these engineering applications have attracted the attention of many researchers to investigate and study the nonlinear vibrations and responses of plates. Feng and Sethna [1] investigated global bifurcations in the motion of parametrically excited, damped thin plates and obtained explicit conditions under which Silnikov homoclinic orbits and chaos can occur. In [2], the method of multiple scales in conjunction with the Galerkin method was used to analyze the nonlinear forced and damped response of a rectangular orthotropic plate subjected to a uniformly distributed harmonic transverse loading. Nonlinear flexural vibrations of a rectangular plate with uniform stretching were studied by Chang et al. [3] for the case when it was harmonically excited with forces acting normal to the midplane of the plate. It was shown that, depending on the spatial distribution of the external forces, the plate can undergo harmonic motions either in one of the two individual modes or in a mixed-mode. Later, Anlas and Elbeyli [4] studied the nonlinear response of rectangular and square metallic plates subject to transverse harmonic excitations. Frequency response curves were presented for both square and rectangular plates for primary resonance of either mode in the presence of a one-to-one internal resonance. In addition, the frequency responses of the nonlinear viscoelastic plates subjected to the subsonic fluid flow and external loads were studied by Younesian and Norouzi [5] for different resonance conditions including non-resonance, primary resonance, super-harmonic resonance and sub-harmonic resonance circumstances. The method of multiple scales was utilized to solve the governing equations and the critical speed of the flow in which the plate can show unstable behavior was obtained. Touze et al. [6] applied the von Karman theory and the method of multiple scales to examine the forced asymmetric nonlinear vibrations of circular plates with a free edge.

The nonlinear vibration of an isotropic cantilever plate with viscoelastic laminate was investigated by Bakhtiari-Nejad and Nazari [7]. Based on Reddy's third-order shear deformation plate theory, Hao et al. [8] investigated the bifurcation and chaotic response of a cantilever functionally graded materials rectangular plate under a combined action of a transverse excitation and temperature field. In addition, Ye et al. [9] analyzed the local and global nonlinear dynamics of a parametrically excited simply supported rectangular symmetric cross-ply laminated composite thin plate using the Galerkin approach and the multiple scales method. Applying the extended Melnikov method in the resonant case, Yao and Zhang [10] investigated the multi-pulse global bifurcations and chaotic

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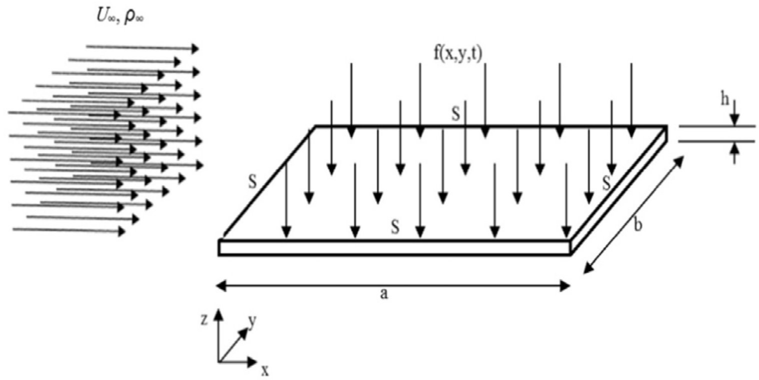

Fig. 1. Schematic representation of the plate subjected to subsonic flow and external excitation.

dynamics of the high-dimension nonlinear system for a laminated composite piezoelectric rectangular plate. The necessary conditions for the existence of the Shilnikov type multi-pulse chaotic dynamics of the laminated composite piezoelectric rectangular plate were analytically obtained. Recently, the nonlinear responses and chaotic motions of plates have been considered by Zhang et al. in a series of papers [11–15].

The nonlinear dynamics of plates in subsonic flow have also been extensively studied in the past years. Applying the Melnikov method, Li et al. [16] studied the chaotic behaviors of a two-dimensional thin panel subjected to subsonic flow and external excitation and obtained the critical parameters for chaos. Based on the potential theory of incompressible flow and the energy method, a two-dimensional simply supported thin panel subjected to external forcing and uniform incompressible subsonic flow was theoretically modeled and the rich dynamical behaviors were presented by Li et al. [17]. The vibrations of the plates interacting with inviscid, incompressible, potential gas flow were analyzed by Avramov et al. [18]. Yao and Li [19] investigated the bifurcation and chaotic motion of a two-dimensional composite laminated plate with geometric nonlinearity subjected to incompressible subsonic flow and transverse harmonic excitation. Li and Yang [20] studied the non-linear dynamical behavior of a cantilevered plate with motion constraints in subsonic flow. Employing the Galerkin method, Li et al. [21] also studied the stabilities and bifurcations of a cantilevered plate with nonlinear motion constraints in an axial subsonic flow.

This paper focuses on research on subharmonic bifurcations and chaotic motions of the nonlinear viscoelastic plates subjected to subsonic flow and external loads. According to the governing equation derived by Younesian and Norouzi [5], the critical conditions for the occurrence of chaotic motions are obtained by Melnikov method. The chaotic features on the system parameters are discussed in detail. The subharmonic Melnikov functions are also computed for the periodic orbits, we obtain that the system can be chaotically excited through infinite subharmonic bifurcations of odd orders. Using the fourth-order Runge–Kutta method, the phase portraits are numerically computed, which agree with the theoretical results.

2. Formulation of the problem

The system treated in this paper is shown in Fig. 1 [5], which illustrates schematic representation of a simply supported plate that is subjected to a subsonic fluid flow and external excitation $f(x, y, t)$ simultaneously. The length, width and thickness of the plate are, respectively, a, b and h. The dimensions of the plate and used coordinates have been demonstrated in Fig. 1. Equation of motion of the plate can be derived and showed in the form [5]:

$$\begin{cases} D\nabla^4 w + ch\dot{w} + \rho h\ddot{w} \\ \quad = f + P + \left(\dfrac{\partial^2 F}{\partial y^2}\dfrac{\partial^2 w}{\partial x^2} - 2\dfrac{\partial^2 F}{\partial x\partial y}\dfrac{\partial^2 w}{\partial x\partial y} + \dfrac{\partial^2 F}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}\right), \\ \nabla^4 F = Eh\left[\left(\dfrac{\partial^2 w}{\partial x\partial y}\right)^2 - \dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}\right]. \end{cases} \tag{1}$$

In this equation E, D, υ, c, ρ and w, are Young's modulus, rigidity, Poisson's ratio, viscous damping, mass density and transverse displacement of the plate respectively. We can define $\nabla^4 = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})^2$ and $D = \frac{Eh^3}{12(1-\upsilon^2)}$. Also P and f represent external pressure distribution and other distributed forces separately, and one can add P to f to obtain total external distributed force. Moreover, F is called the potential function and can be found as

$$\frac{\partial^2 F}{\partial x^2} = N_y, \quad \frac{\partial^2 F}{\partial y^2} = N_x, \quad \frac{\partial^2 F}{\partial x\partial y} = -N_{xy}, \tag{2}$$

in which N_x and N_y are the normal forces per unit length in the x and y directions respectively, and N_{xy} is shear force per unit length. Based on the strain-displacement relations, one can write N_x, N_y and N_{xy} as

$$\begin{aligned} N_x &= \frac{Eh}{1-\upsilon^2}\left[\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\upsilon}{2}\left(\frac{\partial w}{\partial y}\right)^2\right], \\ N_y &= \frac{Eh}{1-\upsilon^2}\left[\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\upsilon}{2}\left(\frac{\partial w}{\partial x}\right)^2\right], \\ N_{xy} &= \frac{Eh}{2(1+\upsilon)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right], \end{aligned} \tag{3}$$

where, u and v are in-plane displacements and are equal zero according to von-Kármán theory. Substituting Eq. (3) into Eq. (2) and then substituting the result into Eq. (1), the equation of motion of the plate can be obtained explicitly in the form [5]

$$\begin{aligned} &D(1 + g_s(\partial/\partial t))\nabla^4 w + ch\dot{w} + \rho h\ddot{w} \\ &\quad = f + P_a + \left[\left(-N_{0x} + \frac{Eh(1 + g_s(\partial/\partial t))}{2a(1-\upsilon^2)}\right.\right. \\ &\quad \times \int_0^a \left(\left(\frac{\partial w}{\partial x}\right)^2 + \upsilon\left(\frac{\partial w}{\partial y}\right)^2\right)dx\left.\right)\left(\frac{\partial^2 w}{\partial x^2}\right) \end{aligned}$$