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Cosmology in one dimension: A two-component model

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We investigate structure formation in a one dimensional model of a matter-dominated universe using a quasi-Newtonian formulation. In addition to dissipation-free dark matter, dissipative luminous matter is introduced to examine the potential bias in the distributions. We use multifractal analysis techniques to identify scale-dependent structures, including clusters and voids. Both dark matter and luminous matter exhibit multifractal geometry over a finite range as the universe evolves in time. We present the results for the generalized dimensions computed on various scales for each matter distribution which clearly supports the bottom-up structure formation scenario. We compare and contrast the multifractal dimensions of two types of matter for the first time and show how dynamical considerations cause them to differ.

1. Introduction

According to galaxy surveys, the universe appears to have large-scale, hierarchical structures up to a certain scale [1,2]. Gravitationally bound collections of luminous galaxies are grouped into clusters and super clusters separated by large voids. As the cosmological principle states that the universe is homogeneous and isotropic at large scales, considerable effort has been made to study the scale at which the universe becomes homogenous [3]. In order to understand the structure of the universe and its associated scale, we need to understand the distribution of dark matter, as it comprises the majority of the matter content of the universe [4]. While the exact nature of “dark matter” is still speculative, observations of the Bullet Cluster strongly imply that dissipation plays a key role in differentiating dark matter from luminous matter [5]. Since the visible galaxies are the only observational tracers, it is important to compare the evolution of both luminous (dissipative) and dark (conservative) matter in a single model where the degree of dissipation can be precisely controlled and investigate the possible bias against the distribution of dark matter. Although we recognize that, in reality, dark matter may behave in a way that is not accounted for in this model, such as macroscopic dissipation, here, following standard practice, we assume that dark matter particles are dissipationless for simplicity. We believe that this model sufficiently captures the essence of hierarchical clustering via weak interactions and demonstrates how the presence of dissipative baryonic matter affects the overall distributions.

Fractal analysis has proven to be a powerful tool in identifying scale-dependent structures as well as in quantifying their textures [6,7]. As fractal analysis does not require a priori knowledge of the mean density, it has been successfully applied in cosmology to find the homogeneity scale both in large scale galaxy surveys and simulations [8,9]. Unlike three-dimensional simulations, a one-dimensional model permits analytical solutions which allow us to maintain fractal fine structures. Therefore, with a one-dimensional model, we can study the non-linear dynamics of the expansion with confidence. In the past, one dimensional models have shown robust scaling ranges, evidence of fractal-like structures [10,11]. Accordingly, it is of wide interest to examine how a one-dimensional model universe with two matter components evolves over time. In particular, to gain information about both high and low density regions of the matter distribution, we employed mass-oriented methods which allow us to investigate the evolution of multifractal spectra D_q, including the negative range of the index q where popular size-oriented methods are known to have difficulty in producing reliable estimates [12]. Moreover, to characterize the evolution of size-dependent dynamical sets, we applied the fixed-k method with various values of k. The use of different values of k allows us to study structures with different sizes. If extrapolated to three-dimensional cosmology, our results clearly demonstrate bottom-up formation, where the clusters with a well-defined fractal dimension spectrum become larger in time. They also indicate that two different multifractal spectra are required to characterize the clusters with the two types of matter. In addition, compared with the dense structures, the voids form more slowly but show no difference for dark matter and luminous matter in terms of their fractal dimensions.

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The main purposes of this work are two-fold: First, we construct a model where we can investigate how the introduction of dissipative matter affects the hierarchical clustering process in a quantitative manner. Second, we demonstrate how both fixed-k and k-neighbor numerical methods can be applied to study the dynamical, scale-dependent fractal structures formed in the clustering process.

2. One-dimensional model

The one-dimensional model was first formulated by Rouet and Feix [13]. Other researchers also have worked on one dimensional models with different coefficients. For details, see the review paper by Miller et al. [10] as well as [14] and [11] for more recent work. In this work, we extend the model to include luminous matter in addition to dark matter. To accomplish this we adopted a simple collision scheme such that luminous matter particles lose energy in interaction with each other. In contrast with dark matter, additional short range forces in luminous matter result in energy loss via radiation, turbulence, etc. Here we lump these effects into an effective inelastic collision between the "luminous" particles. In formulating a one-dimensional model, we embed a set of infinitely large, two-dimensional, parallel sheets of mass with a density m perpendicular to the configuration space. Since the fields generated by the sheets of mass are independent of their position and are parallel to the configuration space, we can confine their motions to an effectively one-dimensional space. Therefore, we represent a sheet by a particle which moves along the configuration space. In order to reduce boundary effects, following typical cosmological simulations [15], we employ periodic boundary conditions which take into account the infinite number of replicas of the mass sheets contained in the original interval $[-L, L)$. While the potential from the infinite number of masses diverges, we can benefit from a technique called Ewald summation. Using this technique, we can isolate the potential which gives rise to the motion of particles by subtracting the background potential [16]. In this way, it can be shown that the total field $E(\chi)$ from the number of particles N in the original interval $[-L, L)$ is

$$E(\chi) = \left[\frac{N}{L}(\chi - \chi_c) + \frac{1}{2}(N_R(\chi) - N_L(\chi))\right] \quad (1)$$

where χ_c is the center of mass of the system and $N_R(L)$ is the number of particles to the right (left) of the position χ within the original interval [16]. Following standard practice, we set up a dynamical equation using Newtonian mechanics with comoving coordinates. During the matter-dominated universe in which structure formation takes place, the universe expands roughly by a scale factor $a(t) \propto (t/t_0)^{2/3}$ [17] for some time unit t where the initial time t_0 may be set to the epoch of recombination, i.e. the beginning of the matter-dominated universe. The comoving coordinate χ is introduced such that the apparent length is kept fixed and the mass density remains constant. The comoving coordinate is related to the original coordinate r by $r = a(t)\chi$. Due to this transformation, we can rewrite the field equation in terms of the comoving coordinate. By introducing a logarithmic time scale T and an appropriate time unit, we obtain

$$\frac{d^2\chi}{dT^2} + \frac{1}{\sqrt{2}}\frac{d\chi}{dT} - \chi = E(\chi, T). \quad (2)$$

This is the signature equation of motion in the RF model, named after Rouet and Feix, and its formulation is fully discussed in their work [13]. With the "friction" coefficient being $\frac{1}{\sqrt{2}}$ in the RF model, we can analytically obtain the crossing time between two particles by solving cubic equations. Thus we can write an event-driven algorithm and minimize the unknown effects often brought in by numerical approximations. In this work, we extend the previous model by introducing luminous matter. In the simulation, luminous matter and dark matter behave identically except at the crossings. When two luminous matter particles approach, they "collide" and lose energy in interaction with each other. We set a velocity-dependent collision coefficient κ analogous to a restitution coefficient. The velocity dependence is given by $\kappa = \exp\left(-c|v_1 - v_2|^{3/5}\right)$ where v_1 and v_2 represent the velocities of two colliding particles. The coefficient c was chosen arbitrarily in the simulation so that the trajectories of luminous matter particles are substantially different from dark matter particles without forcing them to collapse too fast. The luminous particles lose more energy when the velocity difference between the two is large. Initially, the particles are placed near the equilibrium positions which are separated equally in the configuration space.

3. Initial conditions

The primordial potential fluctuation is chosen to replicate the scale-invariant Harrison-Zel'dovich spectrum [18]. In a three-dimensional universe, the spectral index n for the power spectrum is unity which roughly agrees with the estimate from observations [19]. In the one-dimensional case, the spectral index n needs to be three to insure that potential fluctuations are invariant of scale. We randomly assigned initial positions so that the fluctuations around the equilibrium positions follow these statistics. Based on observational estimates, the dark matter to luminous matter ratio is fixed to 4:1 [4]. Accordingly one fifth of the total particles are selected using a random process and designated as luminous matter. The results presented in this work were performed with the total number of particles $N = 100,000$. For simplicity, we chose units such that the original interval length $2L$ is equal to the number of particles N.

4. Simulations

In Fig. 1, we show how the distribution of matter in space evolves over time. For illustrative purposes, this simulation was performed with a smaller number of particles $N = 300$. The initial positions of the particles are shown at the bottom of the figure and, by looking towards the top, we see that the particles coalesce to form clusters. Viewing online one can distinguish the non-dissipative dark matter from the dissipative luminous matter by color. In Fig. 2, we extracted snapshots of the matter distribution at a few different times T in μ-space where the vertical axis represents the velocities of the particles and the horizontal axis the positions. With $N = 100,000$, the system undergoes a similar evolution but on a more massive scale. With $N = 300$, we observe that the nearly homogenous initial distribution evolves into a single cluster towards the end of the simulation. At the core of the cluster, luminous matter appears to be concentrated with dark matter forming a halo around it. We can also see how the energy of the system evolves in Fig. 2. Initially, potential energy (PE) dominates the system. As the total energy (TE) decreases due to the friction term in the equation of motion Eq. (2), the particles begin to pick up kinetic energy (KE). While smaller clusters interact with each other and merge into a larger cluster, potential and kinetic energy exchange. The small, random fluctuations in total energy are due to the collisions between luminous particles.

5. Fractal analysis

By comparing the fractal dimension of the set with the dimension of the embedding space we can estimate the degree of inhomogeneity and complexity. In order to study the formation of the clusters and voids separately, in this work we used the generalized fractal dimensions, for which the well-known box-counting