Chaos, Solitons and Fractals 91 (2016) 262–269

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Generalized intermittent control and its adaptive strategy on stabilization and synchronization of chaotic systems

Cheng Hu*, Juan Yu

College of Mathematics and System Sciences, Xinjiang University, Urumqi, 830046, Xinjiang, P.R. China

ARTICLE INFO

Article history:
Received 10 February 2016
Revised 3 May 2016
Accepted 7 June 2016

Keywords:
Chaotic system
Generalized intermittent control
Adaptive strategy
Stabilization
Synchronization

ABSTRACT

In this paper, the stabilization and synchronization of chaotic systems are investigated by means of intermittent control. At first, a generalized intermittent control and its adaptive strategy are introduced, in which the traditional periodic intermittent control and the aperiodic case are unified. Based on the designed control protocols, by applying comparison principle, the method of piecewise auxiliary function, piecewise analysis technique and the theory of series, some novel and effective criteria are derived to ensure the stabilization and synchronization of chaotic systems. Finally, two numerical examples are provided to verify the theoretical results.

1. Introduction

Chaos, firstly presented by Lorenz in a simple three-dimensional autonomous system [1], is an interesting nonlinear phenomenon and has been extensively investigated in the mathematics, engineering, physics and other related fields [2]. Although it is a very attractive subject, due to its sensitive dependence on initial conditions, chaos was believed in a long time to be neither predictable nor controllable. In 1990, the introduction of OGY control method by Ott. Grebogi Yorke completely denied the viewpoint [3].

During the past few decades, the controlling problem of chaotic systems, including stabilization and synchronization, has been one of the extensive research subjects and many useful control methods are developed such as linear feedback control [4], time-delay feedback control[5], adaptive control [6], fuzzy control [7], sliding mode control [8], impulsive control [9], event-triggered control [10], and intermittent control [11].

Intermittent control, as a new type of discontinuous control, was first introduced to control linear econometric models [12] and has a wide application in manufacturing, transportation and communication. For instance, in communications, intermittent control scheme is usually used as a central means of transmitting information between transmitter and receiver in order to realize synchronization. Intermittent control scheme is composed of work time (or control time) and the rest time in turn, the controller is activated in each work time and is off in the rest time. Compared with impulsive control, intermittent control is easier to be implemented due to it has a nonzero duration called control time. Recently, a periodic case, called periodically intermittent control, has been successfully applied to stabilize and synchronize neural networks [13–20], complex networks [21–25], chaotic systems [26–31]. To cut down control gains, Liu and Chen [32] proposed a centralized adaptive intermittent periodically control and some criteria based on rigorously theoretical analysis were provided to guarantee cluster synchronization of complex networks. In [33], a decentralized adaptive intermittent periodically control was introduced based on pinning strategy to realize synchronization of directed networks.

It is noted that the designed intermittent controller in [13–33] is periodic, as pointed out in [34,35], the periodic intermittent control may be inadequate in the practical application. For example, the generation of wind power is typically aperiodically intermittent. Furthermore, the intermittent control scheme fills the gap between continuous control and impulsive control, while the usual impulsive control requires aperiodic property for its control time period [35]. Therefore, in both application and theoretical analysis, it is of significance to investigate the control problem of nonlinear systems via aperiodic intermittent control. In [34–36], based on pinning control, an aperiodically intermittent control with its adaptive law were introduced to guarantee global synchronization of complex networks.

Notice that the two types of intermittent control, periodic case and aperiodic case, were separately discussed in the previous works. Hence, it is natural to raise the following problems: is there a general framework design to unify periodic and aperiodic intermittent control? If yes, how to design a generalized intermit-

* Corresponding author. Tel.: +8609918582132.
E-mail address: wacheng2003@163.com (C. Hu).

http://dx.doi.org/10.1016/j.chaos.2016.06.004

tent control? Under such control scheme, how to theoretically analyze the stabilization and synchronization of the controlled system, how to establish and obtain the corresponding criteria? However, to the best of our knowledge, they have received little attention up to now. Hence, it is urgent and significant to propose and solve those questions in theory.

Motivated by the above analysis, the aim of this paper is to discuss stabilization and synchronization of chaotic systems by means of a generalized intermittent control protocol and its adaptive strategy. First, as an important preliminary, a generalized intermittent control and its adaptive strategy are designed, in which the traditional periodic intermittent control [13–33] and the aperiodic case [34–36] are included and unified. Secondly, by means of comparison principle, some novel and simple stabilization criteria are derived under the generalized intermittent control with constant control gains. Compared with the recent works [27,28], the proposed technique here is different from their piecewise analysis means and the established criteria are simpler and more practical. Besides, to cut down the control gains, a new adaptive law of intermittent control is proposed to realize stabilization by means of the method of piecewise auxiliary function, piecewise analysis technique and the theory of series. As a similar result, the synchronization of chaotic systems is considered, the corresponding control protocols and the synchronization criteria are also given. Finally, numerical simulations are presented to show the effectiveness of the proposed method.

The rest of the paper is organized as follows. In Section 2, some preliminaries are given. The stabilization and synchronization of chaotic systems are respectively proposed via the designed intermittent control in Sections 3 and 4. In Section 5, the effectiveness and feasibility of the developed methods are shown by a numerical example.

Notations. In this paper, let $R=(-\infty,+\infty)$ be the set of all real numbers, R^n is an n-dimensional real Euclidean space with norm $\|\cdot\|$, $R^{n\times n}$ denotes the set of all $n\times n$ real matrices. For a real matrix A, A^T denotes its transpose. Let Z^+ be the set of all non-negative integers. I denotes the n-dimensional unit matrix.

2. Preliminaries

In this paper, we consider a class of nonlinear chaotic systems described by the following differential equations

$$\dot{x}(t)=Ax(t)+Bf(x(t))+J, \tag{1}$$

where $x(t)=(x_1(t),\cdots,x_n(t))^T\in R^n$ is the state vector, f: $R^n\to R^n$ is a nonlinear vector function, $A\in R^{n\times n}$ and $B\in R^{n\times n}$ are two constant matrices, $J=(J_1,\cdots,J_n)^T$ is a constant vector which may be an external disturbance or the system bias.

To establish our main results, it is necessary to give the following assumption for system (1).

Assumption 1. The nonlinear function f is continuous and satisfies the Lipschitz condition, that is, there is a constant $L>0$ such that for x, $y\in R^n$, one has

$$\|f(x)-f(y)\|\le L\|x-y\|.$$

3. Stabilization problem

In this section, two control schemes will be designed to stabilize nonlinear system (1) to the desired state x^*, which is an equilibrium point of (1) and we assume that it always exists in this paper.

3.1. Intermittent control with constant control gains

To achieve the stabilization, firstly, a generalized intermittent control with constant control gains is designed. The controlled system of (1) can be described by the following form

$$\dot{x}(t)=Ax(t)+Bf(x(t))+J+u(t), \tag{2}$$

where $u(t)$ is a control input which is designed as the following form

$$u(t)=-\hat{d}(t)(x(t)-x^*) \tag{3}$$

with

$$\hat{d}(t)=\begin{cases}d, & t_k\le t\le\delta_k,\\ 0, & \delta_k<t<t_{k+1},\end{cases}$$

here $k\in Z^+$, $d>0$ is a constant, $t_k<\delta_k<t_{k+1}<\delta_{k+1}$.

Remark 1. Recently, intermittent control has been extensively concerned, periodic and aperiodic cases have been separately discussed. To unify them, a general framework design, the generalized intermittent control protocol (3) is introduced in this paper, which contains the traditional periodically intermittent control [13–33] and the aperiodic case [34–36]. Especially, if for all $k\in Z^+$, $t_{k+1}-t_k=T$ and $\delta_k-t_k=\sigma T$, where $T>0$ and $0<\sigma<1$, the generalized intermittent control (3) is reduced to the following periodically intermittent control

$$u(t)=\begin{cases}-d(x(t)-x^*), & t_0+kT\le t\le t_0+(k+\sigma)T,\\ 0, & t_0+(k+\sigma)T<t<t_0+(k+1)T,\end{cases} \tag{4}$$

where $k\in Z^+=\{0,1,2,\cdots,\}$, $d>0$ is the control gain.

To derive the main results, the following assumption is given.

Assumption 2. There exist two finite constants $T>\theta>0$ such that

$$\inf_{k\in Z^+}\{\delta_k-t_k\}=\theta,$$

$$\sup_{k\in Z^+}\{t_{k+1}-t_k\}=T.$$

Theorem 1. *Based on Assumptions 1–2, if*

$$\lim_{m\to+\infty}\sum_{l=1}^{m}\Big[\lambda(t_l-t_{l-1})-2d(\delta_{l-1}-t_{l-1})\Big]=-\infty, \tag{5}$$

then the equilibrium point x^* of system (1) is stabilized under the control scheme (3), where λ is the maximum eigenvalue of the matrix $A+A^T+\varepsilon BB^T+\varepsilon^{-1}L^2I$, $\varepsilon>0$.

Proof. Let $y(t)=x(t)-x^*$, then the error system can be easily derived as the following form

$$\dot{y}(t)=Ay(t)+B(f(y(t)+x^*)-f(x^*))+u(t). \tag{6}$$

Construct the following Lyapunov function

$$V(t)=\frac{1}{2}y^T(t)y(t). \tag{7}$$

Then, by Assumption 1, the right upper Dini derivative of $V(t)$ along the trajectory of the error system (6) can be calculated as follows

$$\begin{aligned}D^+V(t)&=y^T(t)[Ay(t)+B(f(y(t)+x^*)-f(x^*))+u(t)]\\&\le\frac{1}{2}y^T(t)[A+A^T+\varepsilon BB^T+\varepsilon^{-1}L^2I]y(t)-\hat{d}(t)y^T(t)y(t)\\&\le(\lambda-2\hat{d}(t))V(t).\end{aligned} \tag{8}$$

Hence, for all $t\ge t_0$, one has

$$V(t)\le V(t_0)\exp\Big\{\int_{t_0}^{t}(\lambda-2\hat{d}(s))ds\Big\}. \tag{9}$$

Evidently, for any $t\ge t_0$, there exists a nonnegative integer $m\in Z^+$ such that $t_m\le t<t_{m+1}$. According to (9),

$$V(t)\le V(t_0)\exp\Big\{\sum_{l=1}^{m}\int_{t_{l-1}}^{t_l}(\lambda-2\hat{d}(s))ds\Big\}$$