



## Periodic solutions of a spatiotemporal predator-prey system with additional food



Jing Li<sup>a</sup>, Zhen Jin<sup>a,b,\*</sup>, Gui-Quan Sun<sup>b</sup>

<sup>a</sup> Department of Computer Science and Technology, North University of China, Taiyuan, Shan'xi 030051, People's Republic of China

<sup>b</sup> Complex Systems Research Center, Shanxi University, Taiyuan, Shan'xi 030006, People's Republic of China

### ARTICLE INFO

#### Article history:

Received 22 April 2016

Revised 11 June 2016

Accepted 12 June 2016

#### Keywords:

Hopf bifurcation

Time delay

Spatial diffusion

Predator-prey system

### ABSTRACT

In this paper, a spatiotemporal predator-prey system with additional food supplied is investigated. By analyzing eigenvalues of the characteristic equation associated with delay parameter, the conditions of the existence of Hopf bifurcation in one dimension space are obtained. We analyze the properties of bifurcating period solutions based on the normal form theory and the center manifold theorem of partial functional differential equations (PFDEs). Furthermore, numerical simulations confirm the theoretical results. The obtained results may provide some new insights on periodic oscillation in the densities of predator and prey.

© 2016 Elsevier Ltd. All rights reserved.

### 1. Introduction

The role is critical, which interactions between predator and prey play in natural ecosystem, grazing region and agriculture. In order to protect biodiversity and economic benefits, the issue how to effectively control the coexistence of predator and prey is considered. The functional response of predator is defined as the amount of prey caught by a predator in a unit time. Generally, it is appropriate to consider Holling-type functional response. In fact, the species' living environment is not flat, but there are many obstacles which help prey to escape the fate of being eaten by predator, while in turn the amount of prey caught per predator per unit of time is affected. Therefore, taking account for the complexity of prey habitat is necessary in the functional response term [1–7]. In the laboratory systems of *Paramecium aurelia* (prey) and *Didinium nasutum* (predator), Luckinbill showed that the time of coexistence of two species was prolonged by increasing the degree of habitat complexity utilizing methyl cellulose in the Cerophyl medium (nutrient) [8]. The functional response in presence of habitat complexity is

$$f(x) = \frac{a(1-c)x}{1+a(1-c)hx},$$

which is based on the Holling-II  $f(x) = \frac{ax}{1+ahx}$  [9–15]. The term  $c(0 < c < 1)$  represents the degree of habitat complexity which reduces the predation rate.

In nature, most predator may not only eat a kind of food, for example, wheat aphids and scale insect are foods of the coccinellid. Usually, we tend to ignore the influence of additional food (non-prey food) on the predator. The additional food provided to predator have effects on predator-prey systems [16–19]. The additional food supplied decreases the amount of prey captured by predator. Not only the quantity of additional food but also the quality of additional food affect the functional response. The functional response with habitat complexity is further derived as

$$f(x) = \frac{a(1-c)x}{1+\alpha\varepsilon+a(1-c)hx},$$

detailed procedures are described in [20]. In fact, it is valid to consider habitat complexity coupled with supplying additional food to predator in predator-prey model [20].

With the further research of predator-prey systems, many researchers found that the predator can not immediately produce the next generation after eating prey. Therefore, it is more realistic to take into account the gestation delay  $\tau$  ( $\tau > 0$ ) in predator-prey model [21–26]. Since an individual does not always stay in one place, conversely, it diffuses around. However, the previous works did not consider spatial factors [14,16,17,20,27]. Therefore, the reaction-diffusion predator-prey model with delay is able to better reflect the real situation.

\* Corresponding author.

E-mail address: [jinzhn@263.net](mailto:jinzhn@263.net) (Z. Jin).

The structure of this paper is as follows. In Section 2, we derive the characteristic equation of spatiotemporal predator-prey model with additional food supplied, then the existence of the Hopf bifurcation is analyzed. In Section 3, by utilizing the normal form theory and the center manifold theorem, we obtain some properties of Hopf bifurcation. In Section 4, the results of the numerical simulations indicate that time delay can induce periodic changes of the densities of predator and prey in spatiotemporal predator-prey model with additional food supplied. Finally, some conclusions and discussions are given.

## 2. Mathematical modeling and the existence of Hopf bifurcation

### 2.1. Model formulation

For most biological phenomenon, it is more actual to consider latent period [21,23,25,28,29]. The predator may take some time to produce the next generation after consuming prey. For this reason, we need to introduce the time delay into the predator equation. For space environment species live, it is reasonable to consider species diffusion based on the Sahoo model [20]. Moreover, we assume that the population can not pass through the boundary of the domain, and the outside population can not enter this domain. As a result, we have the following system with Neumann boundary conditions in one dimensional space:

$$\begin{cases} \frac{\partial P(x,t)}{\partial t} = rP(x,t)\left(1 - \frac{P(x,t)}{K}\right) - \frac{a(1-c)P(x,t)V(x,t)}{1 + \alpha\varepsilon + a(1-c)hP(x,t)} \\ \quad + d_1 \nabla^2 P(x,t), \\ \frac{\partial V(x,t)}{\partial t} = \frac{\theta a(1-c)(P(x,t-\tau) + \varepsilon)V(x,t-\tau)}{1 + \alpha\varepsilon + a(1-c)hP(x,t-\tau)} - \mu V(x,t) \\ \quad + d_2 \nabla^2 V(x,t), \quad t \geq 0, x \in (0, \pi), \\ \frac{\partial P(x,t)}{\partial x} \Big|_{x=0,\pi} = 0, \quad \frac{\partial V(x,t)}{\partial x} \Big|_{x=0,\pi} = 0, \quad t \geq 0, \\ P(x,t) = \phi_1(x,t) \geq 0, V(x,t) = \phi_2(x,t) \geq 0, \\ (x,t) \in [0, \pi] \times [-\tau, 0], \end{cases} \quad (1)$$

where  $P(x, t)$  represents the density of prey at location  $x$  and time  $t$ ,  $V(x, t)$  the density of predator at location  $x$  and time  $t$ ,  $r$  and  $K$  are intrinsic growth rate and carrying capacity of the prey, respectively,  $a$  is maximum rate of predation,  $h$  is half saturation value of the predator,  $\alpha$  and  $\varepsilon$  measure the quality and quantity of additional food, respectively, the maximum growth rate of the predator is given by  $\theta$ ,  $\mu$  is the natural death of predator,  $d_1, d_2$  are diffusion coefficients.

Assuming  $\phi = (\phi_1, \phi_2)^T \in \mathcal{D} = C([-\tau, 0], X)$ ,  $\tau > 0$  and  $X$  is defined as

$$X = \left\{ (P(x,t), V(x,t))^T : P(x,t), V(x,t) \in W^{2,2}(0, \pi); \frac{\partial P(x,t)}{\partial x} \Big|_{x=0,\pi} = \frac{\partial V(x,t)}{\partial x} \Big|_{x=0,\pi} = 0 \right\}$$

with the inner product  $\langle \cdot, \cdot \rangle$ .

### 2.2. Existence of Hopf bifurcation

At first, in the absence of diffusion and delay, system (1) is corresponding to the following system:

$$\begin{cases} \frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right) - \frac{a(1-c)P(t)V(t)}{1 + \alpha\varepsilon + a(1-c)hP(t)}, \\ \frac{dV(t)}{dt} = \frac{\theta a(1-c)(P(t) + \varepsilon)V(t)}{1 + \alpha\varepsilon + a(1-c)hP(t)} - \mu V(t). \end{cases} \quad (2)$$

The above system has three equilibria including  $E^0(0, 0)$ ,  $E^1(K, 0)$ , and the positive equilibrium  $E^*(P^*, V^*)$ , where

$$P^* = \frac{\mu(1 + \alpha\varepsilon) - \theta a(1-c)\varepsilon}{a(1-c)(\theta - h\mu)},$$

$$V^* = \frac{r(K - P^*)[1 + \alpha\varepsilon + a(1-c)hP^*]}{aK(1-c)}.$$

Considering the biological meaning,  $E^*(P^*, V^*)$  needs to be satisfied the following conditions [20]:

$$(S1) \quad 0 < \varepsilon < \frac{aK(1-c)(\theta - h\mu) - \mu}{\alpha\mu - a\theta(1-c)}, \quad c < 1 - \frac{\mu}{aK(\theta - h\mu)},$$

$$h\mu + \frac{\mu}{aK} < \theta < 1.$$

Let  $p = P - P^*$ ,  $v = V - V^*$ , then system (1) can be translated into the following system:

$$\begin{cases} \frac{\partial p}{\partial t} = r(p + P^*)\left(1 - \frac{p + P^*}{K}\right) - \frac{a(1-c)(p + P^*)(v + V^*)}{1 + \alpha\varepsilon + a(1-c)h(p + P^*)} \\ \quad + d_1 \nabla^2 p, \\ \frac{\partial v}{\partial t} = \frac{\theta a(1-c)(p(t-\tau) + P^* + \varepsilon)(v(t-\tau) + V^*)}{1 + \alpha\varepsilon + a(1-c)h(p(t-\tau) + P^*)} \\ \quad - \mu(v + V^*) + d_2 \nabla^2 v. \end{cases} \quad (3)$$

Defining

$$f^{(1)}(p, v) = r(p + P^*)\left(1 - \frac{p + P^*}{K}\right) - \frac{a(1-c)(p + P^*)(v + V^*)}{1 + \alpha\varepsilon + a(1-c)h(p + P^*)},$$

$$f^{(2)}(p(t-\tau), v(t-\tau), v) = \frac{\theta a(1-c)(p(t-\tau) + P^* + \varepsilon)(v(t-\tau) + V^*)}{1 + \alpha\varepsilon + a(1-c)h(p(t-\tau) + P^*)} - \mu(v + V^*),$$

and for  $i, j = 0, 1, 2, \dots$ , let

$$f_{ij}^{(1)} = \frac{\partial^{i+j} f^{(1)}}{\partial p^i \partial v^j}(0, 0), \quad i + j \geq 1,$$

$$f_{ijl}^{(2)} = \frac{\partial^{i+j+l} f^{(2)}}{\partial p^i(t-\tau) \partial v^j(t-\tau) \partial v^l}(0, 0, 0), \quad i + j + l \geq 1.$$

Further we get

$$\frac{dU(t)}{dt} = D\Delta U(t) + L(U_t) + F(U_t), \quad (4)$$

where  $U = (u_1, u_2)^T$ ,  $p(t, x) = u_1(t, x)$ ,  $v(t, x) = u_2(t, x)$ ,  $D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ . We set  $U_t(\theta) = U(t + \theta)$ ,  $\phi = (\phi_1, \phi_2)^T \in \mathcal{D}$ ,  $\phi(\theta) = U_t(\theta)$ , and  $\theta \in [-\tau, 0]$ .

Let  $L: \mathcal{D} \rightarrow X$  and  $F: \mathcal{D} \rightarrow X$  are given by

$$L(\phi) = \begin{pmatrix} a_{11}\phi_1(0) + a_{12}\phi_2(0) \\ a_{21}\phi_1(-\tau) + a_{22}\phi_2(-\tau) + a_{23}\phi_2(0) \end{pmatrix},$$

where  $a_{11} = f_{10}^{(1)}(0, 0)$ ,  $a_{12} = f_{01}^{(1)}(0, 0)$ ,  $a_{21} = f_{100}^{(2)}(0, 0, 0)$ ,  $a_{22} = f_{010}^{(2)}(0, 0, 0)$ ,  $a_{23} = f_{001}^{(2)}(0, 0, 0)$ , and

$$F(\phi) = \begin{pmatrix} \sum_{i+j \geq 2} \frac{1}{i!j!} f_{ij}^{(1)}(0, 0) \phi_1^i(0) \phi_2^j(0) \\ \sum_{i+j+l \geq 2} \frac{1}{i!j!l!} f_{ijl}^{(2)}(0, 0, 0) \phi_1^i(-\tau) \phi_2^j(-\tau) \phi_2^l(0) \end{pmatrix}.$$

Download English Version:

<https://daneshyari.com/en/article/8254564>

Download Persian Version:

<https://daneshyari.com/article/8254564>

[Daneshyari.com](https://daneshyari.com)