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The effect of time delays on transmission dynamics of schistosomiasis

Chunxiao Ding[a,b], Nana Tao[a,c], Yun Sun[a], Yuanguo Zhu[a,*]

[a] Department of Applied Mathematics, Nanjing University of Science and Technology, Nanjing 210094, Jiangsu, China
[b] School of Mathematical Sciences, Liaocheng University, Liaocheng, 252059, Shandong, China
[c] Department of Mathematics, Kaifeng University, Kaifeng 475004, Henan, China

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ABSTRACT

A 6-dimension dynamical schistosomiasis model incorporating five time delays is established in this paper. Two equilibrium points: a disease free equilibrium and an endemic equilibrium, are calculated respectively. The stability behaviors at the disease free equilibrium are analysed. Both analytical and numerical results are presented that prepatent periods in infection can affect the schistosomiasis transmission significantly. Thus, two effective measures on schistosomiasis prevention and control are obtained: lengthening the prepatent period in susceptible snails, and prolonging the incubation periods in miracidia and cercaria by temperature control or drug restraint. And then, numerical simulations are given to illustrate the validity and effectiveness of the model. At last a discussion is provided about our results and further work.

1. Introduction

Schistosomiasis is a parasitic disease caused by blood flukes of the genus *Schistosoma*, and it is spread by contact with water containing parasites. It has kept spreading across in the tropical and subtropical countries and still remains a threat to public health today. More than 40 million people were treated for schistosomiasis, and at least 261 million people required preventive treatment in 2013. Preventive treatment, which should be repeated over a number of years, will reduce and prevent morbidity. Schistosomiasis transmission has been reported from 78 countries. However, preventive chemotherapy for schistosomiasis, where people and communities are targeted for large scale treatment, is only required in 52 endemic countries with moderate to high transmission, which is in accordance with the data of World Health Organization (WHO) updated in May 2015 [22]. Due to the great harmfulness of schistosomiasis to public society, it is an important issue to understand the biological complexity of schistosomiasis transmission dynamics and to take effective measures in controlling or eliminating its transmission. Mathematical models are powerful tools for insight into the transmission of epidemic disease [10,18]. The first model to investigate transmission dynamics of schistosomiasis was presented in 1965 [15]. Since then, lots of mathematical models have been developed to interpret the interplay of complicated biological principle [1,6]. Especially, a 2-dimension mathematical model, infected human population and infected snails, was one of the classic works [1]. Schistosomiasis models with density dependence and age-structure were given in [6]. The dynamics of growing islets and transmission of schistosomiasis japonica in the yangtze river were studied in [19]. Multi-host system, including two kinds of definitive host and sole intermediate host, is investigated by [4].

These models have provided useful information for understanding the mechanics of schistosomiasis transmission. But as a matter of fact, time delays do exist in schistosomiasis transmission in forms of incubation or maturation period. There are many excellent researches to exploit the time delay influences. A schistosomiasis model was presented with mating structure and time delay [2]. A nonlinear deterministic model for schistosomiasis transmission including delays with two general incidence functions was considered [9]. A two-dimensional system incorporated two time delays was studied, the incubation period of human and snail, which demonstrated that the time delays are harmless for stability of equilibria of the system [3]. All the mentioned investigations concentrate on the time delay influences about humans and snails. The time delays in miracidia and cercaria are ignored. In this paper we focus on the influences caused by time delays in miracidia and cercaria. A 6-dimensional model is presented incorporating humans, snails, miracidia and cercaria with five time delays. And then, dynamical behaviors of the model are explored by implementing the basic methods and theory in delayed differential equation. We obtain the following results:

* Corresponding author.
E-mail addresses: dingchunxiao1987@163.com (C. Ding), nanatao1981@126.com (N. Tao), chinalsy_881220@163.com (Y. Sun), ygzhu@njust.edu.cn (Y. Zhu).

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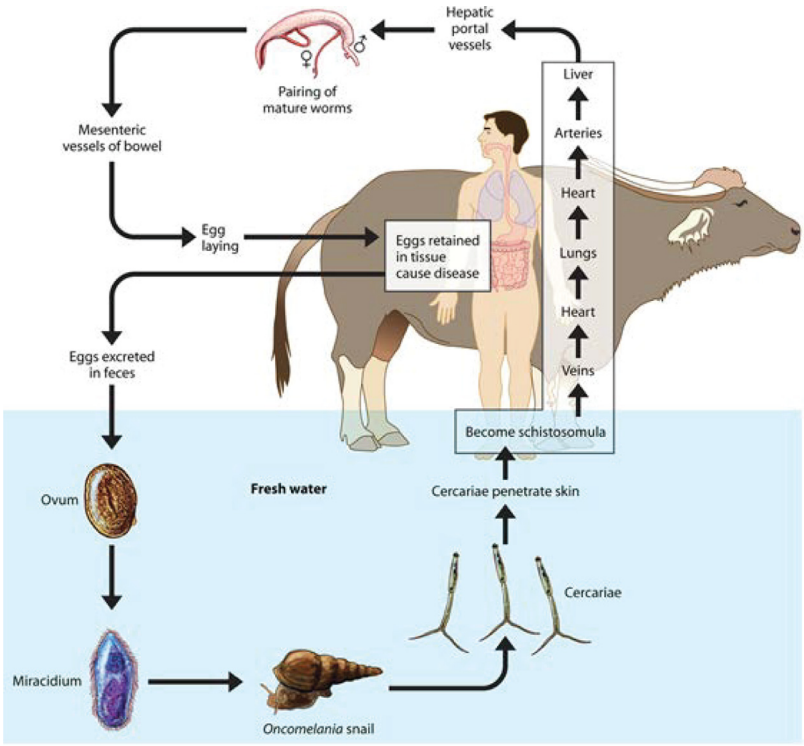

Fig. 1. The life cycle of schistosome (cite from [21]). The parasite development depends on both definitive hosts and intermediate hosts.

- Prolonging the delayed duration of sporocyst in snails by lower water temperature is a powerful measure to restrain the spread of schistosomiasis, due to the time delay in susceptible snails immediately impact the basic reproductive number, which is the key of disease outbreak or die out.
- The time delays in miracidium and cercaria can influence the stability of disease free equilibrium. Only under the condition that the death rate is greater than the transfer rate to the free moving larvae, the disease free equilibrium can keep its locally asymptotical stability;
- It is an effective strategy on schistosomiasis control to lengthen the time delays in miracidium and cercaria in the early stage of spread by drug treatment.

The paper is organized as follows. Section 2 introduces the delayed epidemic model to describe the transmission dynamics of schistosomiasis which incorporates two hosts and two free moving parasite larvae. In Section 3, we calculate the basic reproduction number, and state two equilibriums: a disease free equilibrium and an endemic equilibrium. In Section 4, some stability analyses of the disease free equilibrium are presented for the 6-dimensional schistosomiasis model. We propose some numerical simulations to illustrate the stability results, and summarize some efficient strategies for schistosomiasis prevention and control in Section 5. In the last section, we conclude with a brief discussion about our theoretical and numerical results obtained and further work.

2. Model formulation

The life cycle of schistosome includes several stages: schistosomulum (larval stage), adult schistosome, egg, free-swimming miracidium, sporocyst and cercaria, as shown in Fig. 1. The schistosomulum and adult schistosome rely on the definitive hosts, and the sporocyst can not grow up into miracidium without its one and only intermediate hosts, snails.

We assume that the total population of humans is denoted as $N_h(t)$, which can be divided into two different epidemiological subclasses: susceptible humans and infected humans with sizes $H_s(t)$ and $H_i(t)$ respectively, where s and i represent the subclasses of susceptible and infected respectively. Similarly, the total population of snails is written as $N_s(t)$, which is separated into susceptible subclass $S_s(t)$ and infected subclass $S_i(t)$. Moreover, $M(t)$ and $C(t)$ are referred to the mean spatial density of miracidia and cercaria respectively. The changes of human population are relatively slow in a short time, due mainly to recruitment and death in epidemic areas of schistosomiasis, especially in remote and backward rural countryside in China. Based on these considerations above we assume that the recruitment rate of suspectable human is Λ_h and per capita natural mortality rate of human is d_h in this paper. Similarly, we assume that the snails are recruited at rate Λ_s, and per capita natural mortality rate of snails is d_s. And the natural mortality rate of miracidia and cercaria are denoted as d_m and d_c respectively.

Once a schistosomulum intrudes into its definitive host successfully, it grows up from schistosomulum to adult schistosome with sexual maturity within approximately two months [2]. We use τ_1 to represent the incubation period for schistosome from getting invasion into a susceptible host to excreting faeces with parasite eggs. In some previous works, $H_s(t-\tau_1)e^{-d_h\tau_1}$ signifies the left population of suspectable humans at the present time t due to the nature death of humans in time delay. However, the average life span of human, 70 years, is sufficiently large comparing with the two months time delay, that is, $d_h\tau_1 \approx 10^{-8}$, which yields that $e^{-d_h\tau_1}$ is sufficiently close to 1. So it is reasonable to use $H_s(t-\tau_1)$ to describe the suspectable humans simply and effectively.

In the definitive hosts' body, the larvae develop into adult schistosome. Adult worms live in the blood vessels where the females release eggs. Some of the eggs are passed out of the body in the faeces or urine. They hatch into free-swimming larva (miracidia) in $2-24$ hours at favorable ambient temperature [5]. $H_i(t-\tau_2)$ represents the infected humans with time delay τ_2. The miracidium will survive for several days [11]. During this time, if it can find and invade into a snail under water, it will lose its swimming