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A Mixture Theory formulation for hydraulic or pneumatic transport of solid particles

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Dedicated to Dr. K.R. Rajagopal with admiration and gratitude.

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ABSTRACT

In this paper, we discuss the importance of constitutive modeling of the stress tensors in certain engineering issues, related to the pressure drop and skin friction, encountered in solids transport. To study this problem, we first give a brief account of the formulation of a two-component mixture based on the theory proposed by Massoudi, Rajagopal and co-workers. The mixture consists of a linearly viscous fluid infused with solid particles. The solids particles are modeled as a granular media and it is assumed that the mixture is dense enough so that we can use the theory of interacting continua. The subsequent boundary value problem, flow between two flat plates, is then solved numerically and results for various dimensionless numbers are presented for velocities and volume fraction profiles. The engineering quantities of interests discussed are related to the pressure drop and skin friction at the walls.

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1. Introduction

In a recent paper, Massoudi [62] suggested that one can distinguish at least three stages (phases) of development in the Mixture Theory (the theory of interacting continua): (1) the foundation of the theory; (2) the application of the theory; and (3) the interaction of the theory with experiments. It is no exaggeration to say that in fact there are really only two stages in the development of Mixture Theory, (a) before Rajagopal and (b) after Rajagopal, for he has not only been involved in all three stages, but also his contributions to the three different stages have made Mixture Theory a viable and respectable theory in many areas of engineering science. The first paper appeared in 1981 when Shi et al. [105] studied diffusion of a fluid in a rubber-like non-linear elastic material and over the years various applications of Mixture Theory along with new ideas in constitutive modeling and boundary conditions in diverse areas such as biomechanics, lubrication, asphalts, solids transport, etc., have been used (see [87] for a recent review). The present paper addresses a small fraction of his contribution in this field by highlighting, describing, and applying a Mixture Theory formulation for the flow of solid particles entrained in a viscous fluid.

Multi-component (or multi-phase[1]) flows have become the subject of considerable attention because of their importance in many industrial applications. Flowing mixtures consisting of solid particles entrained in a fluid are relevant to a variety of

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[1] Although the two terms, multi-component or multi-phase, are used interchangeably in the literature, multi-component (or two-component for simple cases) is more accurate. This goes back to the early days of the theory where a two-phase flow usually referred to water and vapor (bubbles), two phases of the same material. However, later on the term was retained even for the cases such as solid particles (for example, coal) and water or air. In the latter case, the term 'two-component' should be used, since coal is not a phase of water (or air).

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Nomenclature

$\mathbf{a}$	acceleration vector
$\mathbf{a}_{\mathbf{vm}}$	relative acceleration between components
a	radius of the sphere
A_i	interaction coefficients, $i = 1–5$
$\mathbf{b}$	body force vector
$\mathbf{D}$	symmetric part of velocity gradient
$\mathbf{f}_I$	interaction force vector
F	volume fraction dependence of drag
$\mathbf{I}$	identity tensor
$\mathbf{L}$	gradient of velocity vector
p	fluid pressure
Re	Reynolds number
$\mathbf{T}$	stress tensor
U_o	reference velocity component
$\mathbf{v}$	velocity vector
$\mathbf{W}$	spin tensor
$\mathbf{x}$	position vector

Greek letters

λ_f	second coefficient of fluid viscosity
μ	first coefficient of fluid viscosity
φ	volume fraction of the solid
ρ	density
ρ_o	reference density
γ	volume fraction of fluid

Subscripts

1, f	referring to the fluid phase
2, s	referring to the solid phase
m	referring to the mixture

Superscripts

T	transpose
$*$	dimensionless quantity

Other symbols

div	divergence operator
∇	gradient operator
tr	trace of a tensor
$\otimes$	outer product
.	dot product

applications such as fluidized beds and pneumatic or hydraulic transport of solid particles. The primary approach for describing and analyzing coal furnaces and combustors has generally been accomplished through experimental studies where empirical correlations are used to describe the complex flows and chemical reactions that occur. Traditionally designers have relied on experiments to produce empirical formulas and correlations. One obvious difficulty with this approach is that, in general, changing the experiment or some of the conditions such as geometry, inlet conditions, particle loading, etc., may change the outcome and hence produce different correlations. This approach is now being augmented with theoretical and computational modeling techniques, which provide the design engineers with the predictive capability and the freedom to choose and change conditions leading to a better design of combustors with higher efficiency, optimum geometry, less pollution, etc. In recent years, many CFD (Computational Fluid Dynamics) codes have been developed. An important issue is the physical models which are embedded in these codes; most of these models are linear constitutive equations.

Historically, two distinct approaches have been used to study multi-component flows, more specifically two-component flows, commonly known as two-phase flows. In the *first* case, the amount of the dispersed phase is so small that the motion of this component does not greatly affect the motion of the continuous phase. This is generally known as the “Dilute phase approach,” sometimes also called the Lagrangian approach. This method has been used extensively in applications such as atomization, sprays, and in flows where bubbles, droplets, and particles are treated as the dispersed phase [106,19,16,97]. In the *second* approach, the two constituents are interacting with each other so that each phase (or component) directly