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Modulational instability and on the existence of rogue wave in electron-ion-positron plasma with kappa distributed electrons

Sailendra Nath Paul, Asesh Roychowdhury*

Department of Physics, Jadavpur University, Kolkata-700032, India

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ABSTRACT

Modulational instability of ion-acoustic wave in an electron-ion-positron plasma is analyzed when the electrons are kappa distributed. Instead of the age old method of reductive perturbation technique we have followed a different methodology put forward by Zakharov, Karpman, Fried and Ichikawa. In this approach the stress is more on physics than the formalism. The nonlinear Schrodinger equation is derived and its two kinds of solution are obtained- Envelope Soliton and Rational Soliton. The stability criteria are established and studied by varying the positron density, temperature and wave number. Over and above we have found both dark- soliton and bright- soliton. An important feature of this method is that we can proceed to the critical case in a much simpler way. It may be added that the rational soliton is not a rogon, but a different form of nonlinear excitation for those values of plasma parameters for which we could test stability.

1. Introduction

The origin of high energy superthermal non-Maxwellian charged particles in astrophysical physical situations is one of the most important unsolved problems till today. Various experimental data indicate that non-Maxwellian particles play the main role in astrophysical situations. In fact, in collisionless plasma the particle distribution can deviate from the Maxwellian one and end to a generalized Lorentzian distribution or simply kappa distribution. In, space and laboratory plasmas often possess an excess population of suprathermal electrons, a fact which is reflected in a power law distribution at high velocity (above the electron thermal speed). This excess suprathermality phenomenon is well modelled by a generalized Lorentzian or kappa distribution [1–4]. The k-distribution was first applied to model velocity distributions observed in space plasmas that were Maxwellian-like at lower velocities, but had a power-law form at higher speeds, [5] and was later applied in a variety of studies, successfully fitting many real space observations [6,7].

Since, the non-Maxwellian distribution in plasma appears to be more appropriate than Maxwellian distribution plasma in space and laboratory plasma environments, authors are attracted with much attention to study the linear and nonlinear propagation of waves in such plasma. It would be very important to study nonlinear wave propagation in electron-ion-positron (e-i-p) plasma considering non-Maxwellian kappa distribution of electrons because such plasma occur in many astrophysical environments such as active galactic nuclei [8,9], pulsar magnetosphere [10], polar regions of neutron stars [11], centre of our galaxy [12], early universe [13,14] and solar atmosphere [15]. The e-i-p plasma has also been produced in laser-solid plasma experiments [16,17]

In past few years, several authors have studied solitary waves and double layers [18–24], modulational instability and envelop soliton [25–33] considering the electron-ion-positron plasma. .But, more interesting results are found on ion-acoustic solitary waves and e-i-p plasma having kappa distribution of electrons. El-Awady et al. [34] have shown that solitons profile becomes shorter and wider in e-i-p plasma with kappa distributed electrons and positrons than that for Maxwellian plasma. In the study of ion-acoustic-waves in magnetized e-p-i plasma with cold ions, kappa distributed electrons and kappa distributed positrons it has been shown by Williams and Kourakis [35] that low value of the spectral index or increase in superthermality leads to lower amplitudes and narrower solitons. In magnetospheric e-i-p plasma with kappa distribution for both electrons and positrons Arshad et al. [36] have investigated the kinetic instability of ion acoustic wave and showed that spectral index plays a key role to destabilize the magnetospheric plasma. However, in recent study it is seen that the properties of modulated waves are significantly modified by the presence of kappa-distributed electrons in plasmas [37]. Effects of suprathermality on the amplitude modulational stability

* Corresponding author at. East Kolkata Centre for Science Education and Research, P-1 B.P.Township, Kolkata-700094, India.

E-mail addresses: drsnpaul@gmail.com (S.N. Paul), arc.roy@gmail.com, asesh_r@phys.jdvu.ac.in (A. Roychowdhury).

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are studied by Sultana [38]. Later, Danehkar et al. [39] have studied electron-acoustic solitary waves in collissionles unmagnetized plasma in presence of a suprathermal electron component. Subsequently, Eslami et al. [41] have studied Modulational instability of electron-acoustic waves in plasmas with superthermal electrons using reductive perturbation technique. It is shown that the presence of superthermal electrons enhances the critical wave number of the modulational instability of electron acoustic waves. Besides, due to the presence of the superthermal electrons, the wave is stable on a vaster region. Moreover, the modulational instability growth rate is lower for a larger population of superthermal electrons. Further, it is shown that increasing values of the relative density ratio $\alpha = n_{e0}/n_{c0}$ shifts the instability domain to lower values of wave number. Modulational instability of ion-acoustic waves in plasma with two-temperature kappa-distributed electrons has been studied by Alinejad et al. [40]. They have shown that the small values of the hot electron populations lead to shrinking the modulation instability region. It is also found the instability growth rate reduces due to the presence of hot electrons. Recently, Ghosh et al. [42] have studied modulational instability of waves in pair ion plasma with kappa distribution of electrons.

On the other hand, in recent years, a new nonlinear phenomenon called as rogon or rogue waves have been studied in plasma. Rogue waves are a singular, rare, high-energy event with very high amplitude that carries dramatic impact. It appears in seemingly unconnected systems in the form of oceanic rogue waves, communication systems, stock market crashes, superfluid helium, Bose-Einstein condensates, opposing currents flows, propagation of acoustic-gravity waves in the atmosphere, atmospheric physics, and plasma physics etc. It is to be pointed out that originally this phenomenon has been studied in water waves in sea [43]. Rogue waves is the name given by oceanographers to the isolated large amplitude waves, that occur more frequently than expected for normal, Gaussian distributed, statistical events. Experiments in multicomponent plasma with negative ions have, recently, reported the evidence of Peregrine solitons of ion-acoustic waves [44]. It is shown that, for a critical concentration of negative ions, slowly amplitude modulated perturbations undergo self-modulation, hence, giving rise to high amplitude localized pulses. The measured amplitude of the Peregrine soliton is three times the nearby carrier wave amplitude, which agrees with the theory and with the numerical solution of the nonlinear Schrödinger equation. Subsequently, numerical investigations have been reported by Moslem et al. [45] for the generation of acoustic rogue waves in dusty plasma composed of negatively charged dust grains and nonextensive electrons and ions. Later, Wang et al. [46] have investigated the solitary waves and rogue waves in plasma featuring Tsallis distribution plasma.

In this regard, it is very important to note that for the analysis on modulational instability of waves from nonlinear Schrodinger (NLS) equation most of the authors used reductive perturbation technique. But, a new approach to modulational instability was put forward by Fried and Ichikawa [47] which was applied to two-temperature plasma by Chakraborty et al. [48]. An important extension of the method was done in this paper, which considered the critical case (in which the nonlinearity is going to vanish in NLS equation) of NLS and a higher order nonlinear Schrodinger equation was deduced. The genesis of such approach goes back to the original observation of Karpman [49], Zakharov and Ostrovsky [50] and Kim et al. [51]. There the authors showed how starting from the basic assumptions of a nonlinerar dispersion relation and a slow variation of the amplitude it is possible to derive the NLS equation. Their analysis was transparent enough to show that in this case of nonlinear waves amplitude modulation automatically implies phase modulation and vice versa. Here it was demonstrated that without making any reference to any particular fluid equation of motion (fluid mechanics, plasma physics or nonlinear optics) it is possible to study modulational instability and derive NLS equation. So starting from these generalized idea and using a discredited from of Fourier analysis Fried and Ichikawa showed how NLS equation can be derived. We have applied such an idea to a situation involving kappa distributed superthermal electrons in an electron-ion-positron plasma.

Since both the kappa-distributed electrons and the positrons have significant effect on nonlinear wave processes in plasma we have here considered the problem of modulational instability in electron-ion-positron plasma in presence of non-Maxwellian kappa distributed electrons. For our present study, we have applied the mathematical technique of Fried and Ichikawa [47]. The nonlinear Schrodinger equation is derived along with the group velocity and dispersion co-efficient. The stability criteria are established and studied by varying the positron density, temperature and wave number. The solutions for ion acoustic Envelope Soliton and Rational Solitons are obtained. Moreover, we have found both dark soliton and bright soliton in electron-ion-plasma. But the standard rogons are not found in this plasma.

2. Formulation

We consider an electron-ion-positron plasma consisting of isothermal cold positive ions and kappa distributed superthermal electrons. The plasma is assumed to be collissionless and unmagnetized. The basic equations are

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \varphi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_p - n_i \tag{3}$$

Here n_i, v_i are ion density and velocity; ϕ, the electrostatic potential; n_e, n_p are the electron and positron density. As per our assumptions the electrons e kappa distributed and we take

$$n_e = \left(1 - \frac{\varphi}{\kappa_e - 3/2}\right)^{-(\kappa_e - 1/2)} = \left(1 - \frac{\varphi}{\kappa_1}\right)^{-\kappa_2} \tag{4}$$

The positron density is given by

$$n_p = \chi \exp(-\sigma_p \varphi) \tag{5}$$

where, $\chi = n_{p0}/n_{e0}, \sigma_p = T_p/T_e$

Assuming the usual wave like behaviour of all physical variable $\sim \exp(i\,k\,x - i\,\omega\,t)$ we get the linear dispersion relation

$$\frac{\omega^2}{k^2} = \frac{1-\chi}{k^2+\alpha} \tag{6}$$

where,

$$\alpha = \kappa_2/\kappa_1 + \chi\sigma_p, \quad \kappa_1 = \kappa_e - 3/2, \kappa_2 = \kappa_e - 1/2 \tag{7}$$

The corresponding group velocity is given by

$$v_g = \frac{(1-\chi)^{1/2}\alpha}{(k^2+\alpha)^{3/2}} \tag{8}$$

3. Formulation of non-linear Schrodinger equation

To start with we assume the existence of a suitable nonlinear dispersion relation

$$\varepsilon(k, \omega, A) = 0 \tag{9}$$