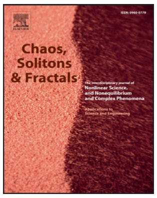
Chaos, Solitons and Fractals 91 (2016) 452–460

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos



Frontiers

Modelling and computing the peaks of carbon emission with balanced growth

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ARTICLE INFO

Article history:
Received 13 April 2016
Revised 6 July 2016
Accepted 9 July 2016

2010 MSC:
90B10
35L65
65K10
65M12
65M60

Keywords:
The interaction model of carbon and biosphere
Discontinuous finite element methods
Existence and uniqueness

ABSTRACT

In this paper, we assume that under the balanced and optimal economic growth path, the economic growth rate is equal to the consumption growth rate, from which we can obtain the ordinary differential equation governing the consumption level by solving an optimal control problem. Then, a novel numerical method, namely a so-called discontinuous Galerkin method, is applied to solve the ordinary differential equation. The error estimation and the superconvergence estimation of this method are also performed. The model's mechanism, which makes our assumption coherent, is that once the energy intensity is given, the economic growth is determined, followed by the GDP, the energy demand and the emissions. By applying this model to China, we obtain the conclusion that under the balanced and optimal economic growth path the CO_2 emission will reach its peak in 2030 in China, which is consistent with the U.S.-China Joint Announcement on Climate Change and with other previous scientific results.

1. Introduction

The anthropogenic greenhouse gas (GHG) emission has risen dramatically during the last few decades, which mainstream researchers believe to be the main cause of climate change, especially the global warming. According to the International Energy Agency (IEA) (2006), to satisfy the energy demand the carbon dioxide emitted mainly by fossil fuels accounts for around 80% of the global emission. Therefore, many efforts have been contributed to capture the future trend of GHG emissions, in hope of providing an estimate of the future warming trend with its influence on the ecological system and human beings, and there are some instructions on how to alleviate or adapt to the global warming.

According to the IEA, China has been one of the countries who emit CO_2 most since 2007, due to its rapid increasing demand for energy and its heavy carbon-content characteristics of energy constitution. Based on the data from the National Bureau of Statistics of China (NBSC, 2011), the energy demand of China, driven by its rapid economic growth, has increased more than from 571.44 million tons of coal equivalent (mtce) in 1978 to 3066.47 mtce by 2009. During 1978–2009, the fossil fuels have accounted for more than 90 percent of energy, and the coal has reached nearly 70% of the total energy demand.

Most governments are reluctant to abate the CO_2 emission, since the abatement may prohibit their pursuit of economic development, especially for the developing countries. Therefore, it is necessary to seek a sustainable pathway for development by alleviating the dependency on energy, which needs the joint efforts of the experts and scholars in all walks of life.

According to [8], the well-known environment Kuznets curve (EKC) implies that the economic development heavily depends on the energy at the beginning of the industrial revolution, and then the dependency weakens when the economy develops into its advanced stage for the reason of technology progress, increasing abatement cost or upgrading industrial structure. Therefore, we can expect the carbon emission to reach its peak in the future if the energy intensity continues dropping in China. In [15], Moon and Sonn (1996) incorporate energy into production function explicitly, and deduce the optimal economic growth rate under the framework of optimal economic growth theory. Zhu and

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http://dx.doi.org/10.1016/j.chaos.2016.07.004
0960-0779/

Wang [30] modify the Moon-Sonn model and obtain the necessary condition of the existence of the inverse U-shaped relationship between the optimal growth rate and the energy intensity, that is, the elasticity of energy in production function should be less than 0.5. Their results show that the peak of energy consumption and carbon emission will appear in 2043 and 2040, respectively. Some other studies have been made to project the CO_2 emission path, such as [1,3,4,7,14,18,20,28]. Additionally, the game theory is a hot research topic in many area, and it is also a good tool for the decision problem described in [2,6,11,12,21–24].

However, most of the existing researches use the specified utility function to simplify the problem, which may not be suitable for describing the realistic world. In this paper, we extend the model in [30] by using a new utility function, in which the minimal consumption level is considered [19]. By means of the Pontryagin maximum principle, we can obtain an ordinary differential equation (ODE) satisfied by the consumption. Our results show that under the balanced and optimal economic growth path, the CO_2 emission will reach its peak in 2030 in China, which is consistent with the U.S.-China Joint Announcement on Climate Change and with other scientific results [16,29,30].

Since the obtained ODE's closed-form solution is difficult to find, we propose a so-called discontinuous Galerkin (DG) method, which is tractable and highly accurate, to solve it numerically. The DG finite element method is a specialized finite element method that utilizes discontinuous piecewise polynomial spaces to approximate the solutions of differential equations with boundary conditions weakly imposed through bilinear forms. Derived from variational principles by integration over local cells, the method is locally mass conservative by construction. In addition, the weak enforcement of boundary conditions leads to the small numerical diffusion and a little oscillations for the DG method. Furthermore, the DG method handles rough coefficient problems and captures the discontinuity in the solution very well by the nature of discontinuous function spaces. Therefore, the DG method is very suitable to our practical economic problem, whose solution may not be smooth efficiently. In recent years, the study of discontinuous Galerkin finite element methods has attracted more and more attentions. See, for instance, [5,9,20,25–27].

Compared with the generalized DG method, we relax the initial approximation of the L^2 orthogonal projection to an arbitrary approximation of $O(h^{r+1})$ accuracy. A novel interpolation plays an important role in our new approach, and we can take its advantage to establish the optimal convergence error estimate of $O(h^{r+1})$. As far as we have known, this result is optimal for hyperbolic problems even for ODEs.

The rest of this paper is organized in the following way. In Section 2, the details of the establishment for the mathematical model are addressed. The projected results for China is given in Section 3, including the pathways of economic growth, energy consumption and carbon emission. The DG method used to solve our ordinary differential equation is briefly discussed in Section 4, and some theoretical results of this method are also discussed in this section. Finally, we give some brief conclusions in Section 5.

2. Dynamic relationship between energy and economy

2.1. Notations

Since there are many mathematical symbols in our model, we first make a list in this subsection, and the details should be explained in the context.

- u: the utility function
- $c(t)$: the goods consumed by each agent at t
- $\bar{c}$: the constant of minimal consumption level
- N_t: the number of agents at t
- ρ: the time preference rate
- N_0: the population at the initial year
- Y_t: the production at t
- A_0: the initial total factor productivity
- ν: the advance speed of A_0
- K_t: the physical capital stocks at t
- E_t: the energy input at t
- L_t: the labor forces at t
- α: the elasticity of capital stocks
- γ: the elasticity of labor forces
- τ: the energy intensity
- θ: the average comprehensive cost of energy
- δ: the depreciation rate of capital stocks

2.2. Dynamic relationship between energy and economy

To describe the realistic market environment better, we use the following utility function in our model

$$u(c_t) = \frac{(c_t + \bar{c})^{1-\sigma}}{1-\sigma},$$

where c_t denotes the goods consumed by each agent in period t, $\bar{c}$ the constant of minimal consumption level, σ the coefficient of relative risk aversion, and $u(\cdot)$ the utility function of individual agent. Compared with the work in [30], this utility function should be more realistic. In this function, the consumption is divided into two parts: one is the luxury consumption and the another one is the basic consumption, which indicates that the consumption that increases utility is luxury consumption, and the basic consumption that ensure the basic life of people can be seen as a constant.

Following the classic assumption of optimal economic growth model, the society consists of many representative agents, who can be taken as adult persons or households. Each agent obtains a certain utility within each period by consuming a certain volume of goods. Thus, the whole society's welfare is the sum of each agent's utility, and the objective of the society is to maximize the sum of the present value of society's future welfare stream over an infinite time horizon:

$$\max_{c_t} \int_0^{\infty} u(c_t) N_t e^{-\rho t} dt, \tag{2.1}$$

where t denotes the time, N_t the agents' number at t, $u(\cdot)$ the utility function of individual agent, c_t the goods consumed by each agent at t, and ρ the time preference rate.

We substitute $u(c_t)$ into the Eq. (2.1) with

$$u(c_t) = \frac{(c_t + \bar{c})^{1-\sigma}}{1-\sigma},$$

the objective function can be rewritten with the following form:

$$\max_{C_t} \int_0^{\infty} N_0^{\sigma} e^{(n\sigma-\rho)t} \frac{(C_t + \bar{C})^{1-\sigma}}{1-\sigma} dt, \tag{2.2}$$

where C_t denotes the gross consumption of the entire society, n is the growth rate of population, and N_0 is the population at the initial year.

Each agent is endowed with a certain physical capital and one unit of labor, and his/her consumed goods are produced with capital, labor and energy. The production function takes the conventional Cobb-Douglas form:

$$Y_t = A_0 e^{\nu t} K_t^{\alpha} E_t^{1-\alpha} L_t^{\gamma}, \ \alpha \in [0, 1], \tag{2.3}$$

where A_0 represents the initial total factor productivity (TFP) with its advance speed ν, K_t the physical capital stocks at t, E_t the energy inputs at t, L_t the labor forces at t, α the elasticity of capital stocks, and γ the elasticity of labor forces. Here, the sum of all elasticities $\alpha + (1-\alpha) + \gamma$ is greater than 1, which represents that