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Stochastic dynamical features for a time-delayed ecological system of vegetation subjected to correlated multiplicative and additive noises

Kang-Kang Wang[a,*], De-Cai Zong[b], Yu Zhou[a], Jian-Cheng Wu[c]

[a] *School of Mathematics and Physics, Jiangsu University of Science and Technology, Zhenjiang 212003, China*
[b] *College of Computer Science and Engineering, Changshu Institute of Technology, Changshu 215500, China*
[c] *State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

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ABSTRACT

In this paper, the generalized potential function, the stationary probability distribution function (SPDF), the mean development time and the mean shrinking time of a time-delayed vegetation growth system induced by cross-correlated internal and external noises are investigated. Our main results are designed to reveal the fact that the resonant phenomenon of the mean first-passage time (MFPT) takes place in the vegetation growth model because of the interaction of different types of noises and time delay. It can inhibit the vegetation system from developing rapidly and reduce the stability of the system by increasing of intensity of multiplicative noise and time delay. Meanwhile, it can produce beneficial effect on maintaining the stability of the vegetation system by increasing the strength of correlated noise. However, it can exert complicated effect on the stability of system by increasing the intensity of additive noise in different cases. During the spread process of the vegetation, it plays a positive role in propelling the development of vegetation by reducing the cross-correlated noise strength and increasing the multiplicative, additive noise intensities and time delay. On the contrary, in the decline process of vegetation system, it plays a crucial part in maintaining the vegetation biomass by increasing the strength of cross-correlated noise and weaken the multiplicative, additive noise intensities and time delay.

1. Introduction

There is a great deal of theoretical and experimental work which can prove that noises play a critical role in the dynamical behaviors of ecological systems [1–4]. Some new progress about the studies performed on the stochastic dynamics in biological systems can be found in Ref [46–49]. These noise interferences can come from either unpredictable complicated interaction in ecological population, or extrinsic noisy natural environment, or man-made disasters. Although internal and external noises appear at the same in a lot of real processes, they often stem from completely different origins and are thought of as independent stochastic processes in most of the previous research [5–7]. In a few situations, whereas, both noises derive from the same origin and thus are dependent on each other. In physics this situation implies that the noises are of the same origin [8–12]. Recently, three types of important noise-induced stability (NES) have drawn the increasing interest from scholars [50–57].

The environmental noises often play significant roles in the dynamics of a great number of ecological systems [13–15]. In particular, noise has all along been thought of as an indispensable element which should be taken into account in the modeling of vegetation dynamics [16,17]. In recent years, the ecological system has drawn the increasing interest from many scholars [18–22]. Holling [23] put forward the point of view that the characters of ecological systems should be defined by two distinct properties: resilience and stability. Klausmeier [24] believed that the formation and maintenance of striped vegetation patterns could be interpreted by means of water-plant dynamics. Shnerb et al. [25] indicated that the threshold-noise process depicts the vegetation spatial organization in the arid zone. The noise-Induced stability in dryland plant ecosystems is brought forward [26–27]. It is discovered that there exists an enhancement of ecosystem resilience and a decrease in the likelihood of catastrophic shifts to the desert state. Guttal and Jayaprakash [28,29] have supplemented the factors of herbivores and forest fires into a version of Shnerb's model of vegetation dynamics, and based on the new stochastic vegetation model. In particular, they investigated the influence of internal or external noise on the bistable vegetation system. Nevertheless, in a more general way, many ecological systems contain the memory. The characteristic of the ecosystem is associated closely with

* Corresponding author.
E-mail address: wkk_com@126.com (K.-K. Wang).

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its past through some memory kernel, and this kernel is called time delay in most cases [30,31]. In fact, time delays are prevalent in the natural world [32–36], and frequently alter substantial dynamics of the ecosystem [37,38]. Furthermore, the fact shows that the combination of noises and time delay exists objectively in the real environment and often changes completely dynamical properties of the system [39]. Recently, Zeng et al. [40,41] have discussed the impact of intrinsic and extrinsic noises on the stochastic resonance and stability in a vegetation model with time delay, which shows that noises and time delay could induce resonance phenomenon in an ecological system of vegetation with a weak periodic signal. They also studied correlated noise-induced stability of the tumor cells system [42], noise and delay-induced catastrophic regime shifts in ecosystems [43]. The significance of time delays in stochastic resonance has also been explored, such as delay-aided stochastic multi-resonances [44], and spatial coherence resonance in delayed Hodgkin–Huxley neuronal networks [45].

In this paper, on the basis of the stochastic vegetation growth model, the properties of the generalized potential function, stationary probability distribution and the mean first passage time for the ecological system disturbed by the additive noise, multiplicative and cross-correlated noises together with the time delay term are investigated. The paper is aimed at providing a comprehensive understanding of the dynamics of the vegetation development system under the impact of all kinds of noises and the time delay. In Section 2, we introduce the stochastic vegetation growth system and analyze the generalized potential function induced by the different types of noises and time delay. In Section 3, the steady state properties for the vegetation growth system in the presence of correlated Gaussian white noises and time delay are discussed in detail. In Section 4, the variation of the mean development and shrinking time between two stable states of the system caused by the noises and time delay are discussed numerically. A detailed conclusion and some description are put forward in the final section.

2. A stochastic vegetation growth model driven by noise terms and time delay

In the light of Shnerb's [25] model of vegetation dynamics, we put forward a mean-field vegetation model for semiarid grazing system which portrays the dynamics of vegetation biomass impacted by many factors such as rainfall, consumption by herbivores and competition for nutrition. The deterministic dynamical equation of the ecological vegetation model can be described as follows:

$$\frac{dB}{dt} = \frac{\rho BR}{1+\alpha B} - \frac{\rho}{B_c}B^2 - \mu\frac{B}{B_0+B}, \tag{1}$$

where B stands for the biomass of vegetation, R represents the mean annual rainfall. ρ is the logistical growth rate for the vegetation biomass, α denotes the consumption rate of water by the biomass, B_c is denoted by the carrying capacity of the biomass, μ is the grazing loss rate, and B_0 is a biomass, at which the loss rate is half maximum. In particular, ρ/B_c is called the crowed effect parameter of the vegetation biomass in a particular area. In fact, the crowed effect coefficient ρ/B_c is always impacted by all kinds of external factors. Therefore, we can rewrite the coefficient ρ/B_c as $\rho/B_c + \xi(t)$, where $\xi(t)$ represents the fluctuation of all external factors, such as the annual rainfall, the humidity, the climate, the forest fire and herbivores. Meanwhile, the factors such as competitions inside the vegetation for the water, soil, sunshine and nutrient could generate an additive noise $\eta(t)$. Furthermore, because of the interaction between different types of external fluctuations and the intrinsic competition of individuals of the vegetation biomass, a cross-correlated noise can be induced spontaneously. On the other hand, it needs a certain time that the vegetation absorbs the water, sunshine and nutrition. Hence, we should introduce a time delay term τ into the vegetation system. Taking into account all above disturbances, we can rewrite Eq. (1) as follows:

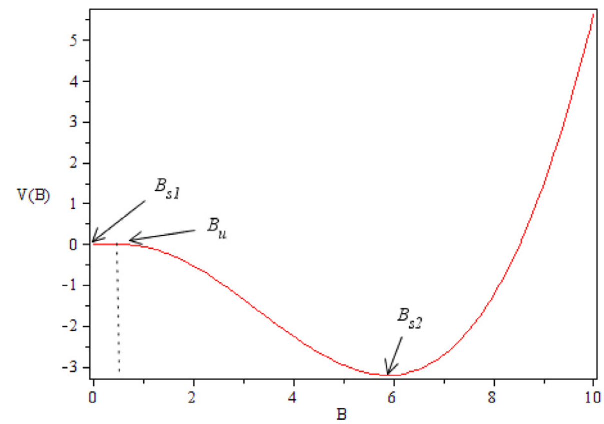

Fig. 1. The bistable potential $V(B)$ with the parameters taking $\rho = 1.0, R = 1.5, B_c = 10.0, \mu = 2.0, \alpha = 0.12, B_0 = 1.0$, including two stable states $B_{s1} = 0$, and $B_{s2} \approx 5.88633$, and one unstable state $B_u \approx 0.45168$.

$$\frac{dB}{dt} = \frac{\rho BR}{1+\alpha B} - \left(\frac{\rho}{B_c} + \xi(t)\right)\cdot B^2 - \mu\frac{B_\tau}{B_0+B} + \eta(t), \tag{2}$$

where B_τ stands for the time delayed variable with $B_\tau = B(t-\tau)\cdot\xi(t)$ and $\eta(t)$ denote Gaussian white noises, whose statistical properties are given as follows:

$$\begin{aligned}
&\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0,\\
&\langle\xi(t)\xi(t')\rangle = 2Q\delta(t-t'),\\
&\langle\eta(t)\eta(t')\rangle = 2M\delta(t-t'),\\
&\langle\xi(t)\eta(t')\rangle = \langle\eta(t)\xi(t')\rangle = 2\lambda\sqrt{QM}\delta(t-t'),
\end{aligned} \tag{3}$$

where Q and M denote the intensities of the multiplicative noise and the additive noise respectively. λ is denoted by the strength of the cross-correlation of the two noise sources. In particular, $-1 < \lambda < 0$ means that the correlation strength between the two noises is negative; $0 < \lambda < 1$ implies that the one between the two noises is positive. The deterministic potential function with respect to Eq. (1) can be written as follows:

$$V(B) = \frac{\rho B^3}{3B_c} - \left(\frac{\rho R}{\alpha} - \mu\right)B + \frac{\rho R}{\alpha^2}\ln(1+\alpha B) - \mu B_0\ln(B_0+B), \tag{4}$$

whose figure in the interval [0, 10] is plotted as follows.

In Fig. 1, we plot the image of the bistable potential $V(B)$ for the determinative vegetation system.

Neglecting the influence of noise terms and time delay term, the fixed points of Eq. (1) which totally dependent on the above parameters are $B_{s1} = 0$ (barren state), and $B_{s2} \approx 5.88633$(sustainable vegetation state), another unstable point is $B_u \approx 0.45168$.

By applying the method of small delay approximation [43], we get the consistent Markovian approximation. Therefore, the logistic Eq. (2) can be rewritten as

$$\frac{dB(t)}{dt} = f_{eff}(B) + g(B)\xi(t) + \eta(t). \tag{5}$$

In which $g(B) = -B^2$, the subscript *eff* stands for the "effective", and the effective coefficient $f_{eff}(B)$ can be described as

$$f_{eff}(B) = \int_{-\infty}^{+\infty}\left[\frac{\rho BR}{1+\alpha B} - \frac{\rho}{B_c}B^2 - \mu\frac{B_\tau}{B_0+B}\right]P(B_\tau, t-\tau|B, t)dB_\tau, \tag{6}$$