



## Complex dynamics and bifurcation analysis of host–parasitoid models with impulsive control strategy



Jin Yang<sup>a,\*</sup>, Sanyi Tang<sup>b</sup>, Yuanshun Tan<sup>a</sup>

<sup>a</sup> Department of Mathematics, Chongqing Jiaotong University, Chongqing 400074, P.R. China

<sup>b</sup> College of Mathematics and Information Science, Shaanxi Normal University, Xi'an, 710062, P.R. China

### ARTICLE INFO

#### Article history:

Received 23 September 2015

Revised 9 July 2016

Accepted 11 July 2016

#### Keywords:

Host–parasitoid model  
Impulsive control strategy  
Periodic solution  
Coexistence  
Bifurcation

### ABSTRACT

In this paper, we propose and analyse two type host–parasitoid models with integrated pest management (IPM) interventions as impulsive control strategies. For fixed pulsed model, the threshold condition for the global stability of the host–eradication periodic solution is provided, and the effects of key parameters including the impulsive period, proportionate killing rate, instantaneous search rate, releasing constant, survival rate and the proportionate release rate on the threshold condition are discussed. Then latin hypercube sampling /partial rank correlation coefficients are used to carry out sensitivity analyses to determine the significance of each parameters. Further, bifurcation analyses are presented and the results show that coexistence of attractors existed for a wide range of parameters, and the switch-like transitions among these attractors indicate that varying dosages and frequencies of insecticide applications and numbers of parasitoid released are crucial for IPM strategy. For unfixed pulsed model, the results show that this model exists very complex dynamics and the host population can be controlled below ET, and it implies that the modelling methods are helpful for improving optimal strategies to design appropriate IPM.

© 2016 Elsevier Ltd. All rights reserved.

### 1. Introduction

Integrated pest management (IPM), usually consists of biological, cultural, chemical control tactics or their combinations, is a long-term control strategy with aim of reducing pest populations to tolerable levels acceptable to the public [1–3]. It is confirmed by experimental results that IPM is more effective than a single control strategy [4–6]. In order to show how IPM strategy can be realized, two important concepts containing economic injury level (EIL) and economic threshold (ET) are introduced (for details see [1]).

In a natural world, the major impediments in pest control for agricultural scientists include (a) how to determine the dosage, period and frequency of IPM applications? (b) when should be an IPM strategy applied in order to prevent the pest population reaching EIL? (c) what is the optimal control strategy for pest control? To answer these questions, mathematical modelling plays a key role enable us to estimate and predict population densities, which would tell us when ET can be reached for pest populations and an IPM strategy should be applied immediately [7–9]. Based on these purposes, numerous of research papers with IPM have appeared

on this topic, most of which have focused on developing and investigating continuous predator–prey models concerning impulsive control strategy [10–17]. From biological significance, the results obtained from these studies show that dosages and frequencies of insecticide applications and timing of natural enemy released can be determined which are benefit for designing pest control tactics.

However, a common feature among many insect species is that they have no overlap between successive generations and so their population evolves in discrete generations. In this case, mathematical modelling of discrete host–parasitoid models, which are usually described by difference equations, are being used to investigate the population dynamics of a species more realistic rather than those of continuous-time models [18–23]. Note that the parasitoid intergenerational survival rate which is affected by several different factors has never been modelled explicitly, these factors often include immigration from outside the local area, parasitoid overwintering survival and parasitism of an alternate insect pest, etc. [24–27]. With this scenario the discrete host–parasitoid models with IPM strategies were firstly proposed and analysed by Tang [28]. In particular, the classical Nicholson–Bailey models incorporating parasitoid intergenerational survival rate with impulsive control strategies were considered based on the assumption that the host population would grow infinity. However, in reality the host population cannot growth toward infinity because of resource limitation,

\* Corresponding author.

E-mail address: [seehom@126.com](mailto:seehom@126.com) (J. Yang).

predation and intra-specific competition, etc. This indicates that the carrying capacity of the host population should be taken into account.

The main purpose of the present paper is to investigate the dynamics of Holling type II host-parasitoid models concerning carrying capacity with IPM strategies. Firstly, we propose a Holling type II host-parasitoid model with IPM applied at different fixed periods. The threshold condition for the stability of the host-eradication periodic solution is provided, then sensitivity analyses are carried out to determine the significance of each parameters. The results show that the dynamics of the proposed model with fixed moments are very complex including period-doubling bifurcation, period-halving bifurcation, chaotic crisis and so on. Coexist attractors are also detected and have been confirmed by basins of attraction, it indicates that the final state of the host population depends on its initial conditions. In addition, the switch-like behaviour is also observed under small random perturbations.

In practice, it suggests that an IPM strategy should only be applied once the states of the model reach a prescribed given threshold. Based on this ideal, we propose and analyse a Holling type II host-parasitoid model with state-dependent feedback control. Compared to the model with fixed moments, it is pointed out that the required dosages and frequencies of insecticide applications are reduced if the density of the host population is only kept below ET rather than being eradicated. Further, bifurcation analyses show that this model exhibits very rich dynamical behaviour including period-adding bifurcation, period-decreasing bifurcation, coexistence of attractors, etc. The results reveal that the host population can be controlled below ET, and the modelling methods can help us to improve optimal strategies in the design of appropriate IPM strategies.

## 2. Holling type II host-parasitoid model with impulsive effects

### 2.1. Model formation

Recently, the classic Nicholson-Bailey models with impulsive control strategies have been proposed and analysed [28], and we note that the solutions of the Nicholson-Bailey model may tend to infinity. However, in a natural world, it is more reasonable for the pest population to tend to carrying capacity rather than infinity due to resource limitation, predation, or other intervention mechanisms. This implies that the host-parasitoid models involving carrying capacity will provide a more natural description for mathematical modelling of real world phenomenon. These assumptions result in the following Holling type II functional response host-parasitoid model [29]:

$$\begin{cases} H_{n+1} = H_n \exp \left[ r \left( 1 - \frac{H_n}{K} \right) - \frac{aTP_n}{1 + aT_h H_n} \right], \\ P_{n+1} = H_n \left[ 1 - \exp \left( -\frac{aTP_n}{1 + aT_h H_n} \right) \right] + \lambda P_n. \end{cases} \quad (1)$$

where  $H_n$  and  $P_n$  are the density of hosts and parasitoids in generation  $n$  ( $n = 0, 1, 2, \dots$ ).  $r$  is the intrinsic growth rate of the host population without parasitoids,  $K$  is the carrying capacity,  $a$  denotes the instantaneous search rate,  $T$  is the total time initially available for search, i.e., the total time the hosts are exposed to parasitoids,  $T_h$  is the handling time, i.e., the time between host being encountered and search being resumed.  $\lambda \geq 0$  is the density-independent survival of the parasitoid at generation  $n$  which represents immigration from outside the local area, releases of biological control agents, and increased growth rate of the parasitoid caused by parasitism of an alternate insect pest [24–27].

In order to investigate the dynamics of system (1) with IPM strategies, we extend system (1) by introducing periodic control strategies such as periodic releasing natural enemies and spraying

pesticide at a fixed period. Note that impulsive reduction of the pest population is possible by trapping the pests and/or by poisoning them with chemicals, while impulsive increase of the parasitoid population can be achieved by releasing natural enemies into the field [28]. These modifications lead to the following Holling type II functional response host-parasitoid model with IPM control strategies:

$$\begin{cases} H_{n+1} = H_n \exp \left[ r \left( 1 - \frac{H_n}{K} \right) - \frac{aTP_n}{1 + aT_h H_n} \right], \\ P_{n+1} = H_n \left[ 1 - \exp \left( -\frac{aTP_n}{1 + aT_h H_n} \right) \right] + \lambda P_n, \\ H_{qk^+} = (1 - q_1)H_{qk}, \\ P_{qk^+} = (1 + q_2)P_{qk} + \tau, \end{cases} \quad n = 0, 1, 2, \dots, \quad k = 1, 2, \dots, \quad (2)$$

where  $q$  is a positive integer and denotes the period of the impulsive effect, and it indicates that control tactics should be applied when  $n$  is an integer multiple of  $q$ .  $q_1$  is the killing rate due to applications of insecticide,  $q_2$  is a proportion representing the releasing rate of the parasitoid at generation  $qk$ , and  $\tau$  is a releasing constant which does not depend on the density of parasitoids. Moreover,  $H_{qk}$  and  $P_{qk}$  are the densities of the host and parasitoid at generation  $qk$  before the impulsive effects, while  $H_{qk^+}$  and  $P_{qk^+}$  denote the densities of the host and parasitoid at generation  $qk$  after the impulsive effects. For convenience, the initial conditions are denoted as  $(H_0^+, P_0^+) = (H_0, P_0)$  and the initial densities are taken after an impulsive effect. It is noted that a very special case of system (2) was studied by Tang et al. [28], who did not consider the effects of carrying capacity and Holling type II functional response.

### 2.2. Host-eradication periodic solution and sensitivity analysis

In order to investigate the existence and stability of host-eradication periodic solution of system (2), we first consider the following subsystem:

$$\begin{cases} P_{n+1} = \lambda P_n, \quad n = 0, 1, 2, \dots, \\ P_{qk^+} = (1 + q_2)P_{qk} + \tau, \quad k = 1, 2, \dots, \\ P_0^+ = P_0. \end{cases} \quad (3)$$

Note that system (3) is a periodic system, so the solution  $P_n$  can be defined at impulsive subinterval  $n \in [qm^+, q(m+1))$  with  $m = 0, 1, 2, \dots$ , and in this interval  $n = qm^+$  implies that after an impulsive effect the density of parasitoids is taken as the initial value.

From system (3), there exists a periodic solution over the interval  $[qm^+, q(m+1))$ , and the complete expression for the host-eradication periodic solution of system (2) can be got as follows

$$(0, P_n^*) = (0, \lambda^{n-qm} \hat{P}) = \left( 0, \frac{\tau \lambda^{n-qm}}{(1 - (1 + q_2)\lambda^q)} \right), \quad (4)$$

where  $1 - (1 + q_2)\lambda^q > 0$ .

The solution of system (3) enjoys the following property.

**Lemma 1.** *If  $1 - (1 + q_2)\lambda^q > 0$  holds, then system (3) has a positive periodic solution  $P_n^*$  and for every solution  $P_n$  of (3) we have  $|P_n - P_n^*| \rightarrow 0$  as  $n \rightarrow \infty$ .*

**Theorem 2.** *Let  $(H_n, P_n)$  be any solution of system (2). Then the host-eradication periodic solution  $(0, P_n^*)$  is globally asymptotically stable in the first quadrant provided that*

$$rq < \ln \left( \frac{1}{1 - q_1} \right) + \frac{aT\tau(1 - \lambda^q)}{(1 - (1 + q_2)\lambda^q)(1 - \lambda)}. \quad (5)$$

The proof of Theorem 2 is similar to that shown in [28] and we omit it.

Download English Version:

<https://daneshyari.com/en/article/8254647>

Download Persian Version:

<https://daneshyari.com/article/8254647>

[Daneshyari.com](https://daneshyari.com)