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Deng entropy

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ABSTRACT

Dempster Shafer evidence theory has been widely used in many applications due to its advantages to handle uncertainty. However, how to measure uncertainty in evidence theory is still an open issue. The main contribution of this paper is that a new entropy, named as Deng entropy, is presented to measure the uncertainty of a basic probability assignment (BPA). Deng entropy is the generalization of Shannon entropy since the value of Deng entropy is identical to that of Shannon entropy when the BPA defines a probability measure. Numerical examples are illustrated to show the efficiency of Deng entropy.

1. Introduction

How to measure the uncertainty has attracted much attention [1,2]. A lot of theories have been developed, for example, probability theory [3], fuzzy set theory [4], Dempster-Shafer evidence theory [5,6], rough sets [7], generalized evidence theory [8] and D numbers [9].

Dempster-Shafer theory evidence theory [5,6] is widely used in many applications such as decision making [10–13], supplier management [14–16], pattern recognition [17], risk evaluation [18,19] and so on [20]. However, some open issues are not well addressed. First, conflicting management should be taken into consideration when evidence highly conflicts with each other, since it may obtain the count-intuitive results [8,21]. Second, the assumption of evidence independent with each other is not satisfied in real application [22]. Third, how to generate the basic probability assignment (BPA) is still an open issue [23,24]. Finally, how to measure the uncertain degree of BPA is not yet solved, which is the aim of this paper. In Dempster-Shafer evidence theory, the uncertainty simultaneously contains nonspecificity and discord [25] which are coexisting in a basic probability assignment function (BPA). Several uncertainty measures [26], such as AU [27,28], AM [25], have been proposed to quantify such uncertainty in Dempster-Shafer theory. Some computing algorithms are also presented to obtain the uncertain degree [29]. What's more, five axiomatic requirements have been further built in order to develop a justifiable measure. These five axiomatic requirements are range, probabilistic consistency, set consistency, additivity, subadditivity, respectively [30]. However, existing methods are not efficient to measure uncertain degree of BPA [31–33].

Since firstly proposed by Clausius in 1865 for thermodynamics [34], various types of entropies are presented, such as information entropy [35], Tsallis entropy [36], nonadditive entropy [37]. Information entropy [35], derived from the Boltzmann-Gibbs (BG) entropy [38] in thermodynamics and statistical mechanics, has been an indicator to measure uncertainty which is associated with a probability density function (PDF). In this paper, a new entropy, named as Deng entropy, is proposed to handle the uncertain measure of BPA. Deng entropy can be seen as the generalized Shannon entropy. When the BPA is degenerated as probability distribution, Deng entropy is degenerated as Shannon entropy.

The paper is organized as follows. The preliminaries Dempster-Shafer evidence theory and entropy are briefly introduced in Section 2. Section 3 presents Deng entropy and gives its important theoretical features. Some numerical examples are illustrated in Section 4 to show the efficiency of Deng entropy. Finally, this paper is concluded in Section 5.

2. Preliminaries

In this section, some preliminaries are briefly introduced.

2.1. Dempster-Shafer evidence theory

Dempster-Shafer evidence theory has many advantages to handle uncertain information. Some basic concepts in D-S theory are introduced as follows [5,6].

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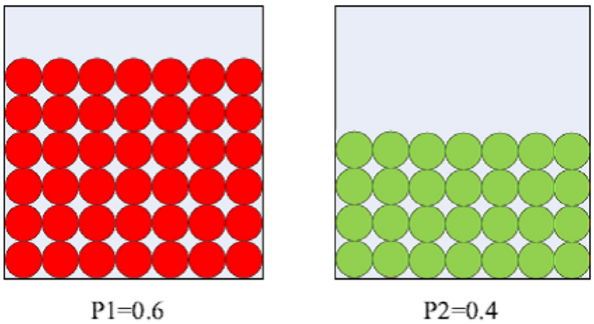

Fig. 1. A game of picking ball which can be handled by probability theory. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

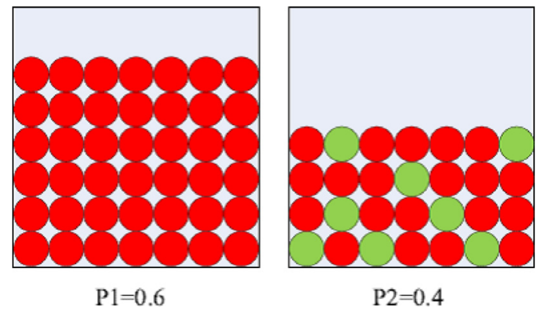

Fig. 2. A game of picking ball where probability theory is unable but D-S theory is able to handle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Let X be a set of mutually exclusive and collectively exhaustive events, indicated by

$$X = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_{|X|}\} \tag{1}$$

where set X is called a frame of discernment. The power set of X is indicated by 2^X, namely

$$2^X = \{\emptyset, \{\theta_1\}, \cdots, \{\theta_{|X|}\}, \{\theta_1, \theta_2\}, \cdots, \{\theta_1, \theta_2, \cdots, \theta_i\}, \cdots, X\} \tag{2}$$

For a frame of discernment $X = \{\theta_1, \theta_2, \cdots, \theta_{|X|}\}$, a mass function is a mapping m from 2^X to [0, 1], formally defined by [5,6]:

$$m: \quad 2^X \to [0, 1] \tag{3}$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad and \quad \sum_{A \in 2^X} m(A) = 1 \tag{4}$$

A mass function is also called a basic probability assignment (BPA). Assume there are two BPAs indicated by m_1 and m_2, the Dempster's rule of combination is used to combine them as follows [5,6]:

$$m(A) = \begin{cases} \dfrac{1}{1-K} \displaystyle\sum_{B \cap C = A} m_1(B) m_2(C)\,, & A \neq \emptyset; \\ 0\,, & A = \emptyset. \end{cases} \tag{5}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{6}$$

Note that the Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $K < 1$.

Here is an example to show the efficiency to model uncertainty of evidence theory. As shown in Fig. 1, there are two boxes. There are red balls in the left box, and green balls in the right box. The number of balls in each box is unknown. Now, a ball is picked randomly from these two boxes. We know that the left box is selected with a probability $P1 = 0.6$, and the right box is selected with a probability $P2 = 0.4$. Based on probability theory, it can be obtained that the probability of picking a red ball is 0.6, the probability of picking a green ball is 0.4, namely $p(R) = 0.6$, $p(G) = 0.4$.

Now, let us change the situation, as shown in Fig. 2. In the left box, there are still only red balls. But in the right box, there are not only red balls but also green balls. The exact number of the balls in each box is still unknown and the ratio of red balls with green balls is completely unknown. We know that the left box is selected with a probability $P1 = 0.6$, and the right box is selected with a probability $P2 = 0.4$. The question is what the probability that a red ball is picked. Obviously, due to the lack of adequate information, $p(R)$ and $p(G)$ cannot be obtained in this case. Facing the situation of inadequate information, probability theory is incapable. However, if using D-S theory to analyze this problem, we can obtain a BPA that $m(R) = 0.6$ and $m(R, G) = 0.4$. In the framework of evidence theory, the uncertainty has been expressed in a reasonable way.

2.2. Existing entropy and open issue

The concept of entropy is derived from physics [34]. In thermodynamics and statistical mechanics, the entropy often refers to Boltzmann-Gibbs entropy [38]. According to Boltzmann's H theorem, the Boltzmann-Gibbs (BG) entropy of an isolated system S_{BG} is obtained in terms of the probabilities associated the distinct microscopic states available to the system given the macroscopic constraints, which has the following form

$$S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i \tag{7}$$

where k is the Boltzmann constant, W is the amount of distinct microscopic states available to the isolated system, p_i is the probability of microscopic state i satisfying $\sum_{i=1}^{W} p_i = 1$. Equal probabilities, i.e. $\forall i$, $p_i = 1/W$, is a particular situation. In that situation, BG entropy has the following form

$$S_{BG} = k \ln W \tag{8}$$

In information theory, Shannon entropy [35] is often used to measure the information volume of a system or a process, and quantify the expected value of the information contained in a message. Information entropy, denoted as H, has a similar form with BG entropy

$$H = -\sum_{i=1}^{N} p_i \log_b p_i \tag{9}$$

where N is the amount of basic states in a state space, p_i is the probability of state i appears satisfying $\sum\limits_{i=1}^{W} p_i = 1$, b is base of logarithm. When $b = 2$, the unit of information entropy is bit. If each state equally appears, the quantity of H has this form

$$H = \log_2 N \tag{10}$$

In information theory, quantities of H play a central role as measures of information, choice and uncertainty. For example, the Shannon entropy of the game shown in Fig. 1 is $H = 0.6 \times \log_2 0.6 + 0.4 \times \log_2 0.4 = 0.9710$. But, it is worthy to notice that the uncertainty of this game shown in Fig. 2 cannot be calculated by using the Shannon entropy.

According to mentioned above, no matter the BG entropy or the information entropy, the quantity of entropy is always associated with the amount of states in a system. Especially, for the case of equal probabilities, the entropy or the uncertainty of a system is a