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Hybrid synchronization of heterogeneous chaotic systems on dynamic network

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ABSTRACT

For researching the hybrid synchronization of heterogeneous chaotic systems on the complex dynamic network, there are two important issues to be discussed and analyzed. One is how to build a dynamic complex network which the connection between nodes is dynamic. Another is comparing and analyzing the synchronization characteristics of heterogeneous chaotic systems on the dynamic and static complex network. In this paper, the theoretical analysis and numerical simulation are implemented to study the synchronization on different dynamic and static complex networks. The results indicate it is feasible to realize the hybrid synchronization of heterogeneous chaotic systems on the complex dynamic network.

1. Introduction

From proposing the concept of complex network to present, the complex network has entered into a prosperous stage. It has been wildly applied into various large-scale networks, such as traffic networks [1,2], internet networks [3–5], biological networks [6,7] and social networks [8,9] etc. However, researching the synchronization on complex networks is a hot direction. In reality, some networks exist synchronization phenomenon, such as power networks [10,11], social networks [12] and biological networks [13,14] etc. Therefore, the synchronization of complex networks proposes important researching values. Nowadays, many scholars have made a deep research about synchronization. For example, Wang etc. put forward a sufficient condition of satisfying synchronization on the scale-free network and small-world network [15]. Lv etc. make a study on the time-varying coupling network model and propose a condition of satisfying synchronization [16]. Xu etc. investigate adaptive synchronization of stochastic time-varying delay dynamical networks with complex-variable systems [17]. A. Rosich etc. discuss the design of communication network topology in a distributed diagnosis system for improving the fault detection and isolation performances [18]. Li etc. investigate the Uncertainty Quantification (UQ) of xponential Synchronization (ES) problems for a new class of Complex Dynamical Networks (CDNs) with hybrid Time-Varying Delay (TVD) and Non-Time- Varying Delay (NTVD) nodes by using coupling Periodically Intermittent Pinning Control (PIPC) which has three switched intervals in every period [19–21]. LIN etc. consider the chaos multiscale-synchronization between two different Fractional-order Hyper chaotic System (FO-HCS)s have been investigated [22,23]. Rakkiyappan etc. research the problem of exponential synchronization of complex dynamical networks with Markovian jumping parameters using sampled-data and Mode-dependent probabilistic time-varying coupling delays is investigated [24,25]. Natarajan Sakthivel etc. investigate a synchronization problem for complex dynamical networks with additive time-varying coupling delays via non-fragile control is investigated [26,27].

However, the connection among real network nodes is dynamical. For instance: on the human social networks, some people's social connection can be broken as a result of long-term business trip or going abroad, or increased by making friends; on the electric power networks, some nodes can be disconnected as a result of long-term disrepair or fault, or rebuilt by maintenance. The above instances all illustrate that the topological structure among network nodes can be randomly changed. Therefore, the following question is how to build the dynamic complex network in conformity with the reality. Although researchers have put forward constructing complex dynamic network by finite modal or mutually switching topology to obtain dynamic. However, such complex dynamic network is stationary and predictable rather than random. So how to build a random and unpredictable complex dynamic system, which conforms to the real life? This is an unavoidable and valuable problem. This paper mainly researches the heterogeneous chaotic systems hybrid synchronization on the dynamic complex network. One is constructing dynamic complex networks in conformity with the real life, where the topology is random and unpredictable. Another is realizing the hybrid synchronization of

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heterogeneous chaotic systems on the dynamic and static complex networks, and analyzing their synchronization characteristics. Through the theoretical analysis and numerical simulation, this paper successfully indicates it is feasible to achieve the hybrid synchronization of heterogeneous chaotic systems on dynamic network. In addition, the results provide a theoretical basis for chaotic mask and chaotic parameter regulation on the secure communication.

The rest of the paper is organized as follows: In Section 2, some basic concepts and the problem are described, In Section 3, the local synchronization of hybrid state is investigated. In Section 4, the global synchronization of hybrid state is investigated also. In Section 5, numerical simulations on different networks are implemented. Finally, conclusions are given in Section 6.

2. Problem description

Suppose two nonlinear dynamic systems are described as follows:

$$\dot{X} = f(X) \quad (1)$$

$$\dot{Y} = g(Y) \quad (2)$$

Here, $X \in \Re^n$, $Y \in \Re^n$ is state vector and f: $B \subseteq \Re^n \to \Re^n$ g: $B \subseteq \Re^n \to \Re^n$ is the non-linear smooth vector field. Hence, the dynamic network systems can be described by:

$$\dot{X}_i = f(X_i) + c\sum_{j=1}^{N} C_{ij}(t)\phi(t, X_j), \quad (i = 1, 2, 3, \ldots, N) \quad (3)$$

$$\dot{Y}_i = g(Y_i) + c\sum_{j=1}^{N} C_{ij}(t)\phi(t, Y_j), \quad (i = 1, 2, 3, \ldots, N) \quad (4)$$

Here, $X_i = (x_{i1}, x_{i2}, \ldots x_{in})^T \in B$ and $Y_i = (y_{i1}, y_{i2}, \ldots y_{in})^T \in B$ jointly represent state vector of the ith vertex. ϕ: $B \in \Re^n$ is unknown nonlinear smooth diffusion coupling functions. $\sum_{j=1}^{N} C_{ij}\phi(t, X_j)$ and $\sum_{j=1}^{N} C_{ij}\phi(t, Y_j)$ synthetically characterize the coupling term of i th vertex which has been affected by other vertices through links between each other at t moment. The network is composed of N vertices, i and j respectively represents two different vertices on the network, and every vertex is identical just like the description of Eqs. (3) and (4). c is fixed coupling strength. $C_{ij} \in \Re^{N\times N}$ is the coupling configuration matrix which represents coupling strength and topological structure on the network, and it also can be called as outer-coupling matrix. Here, C_{ij} is defined as below: if the ith vertex and the jth vertex have a connection, $C_{ij} \neq 0 (i \neq j)$; otherwise, $C_{ij} = 0 (i \neq j)$. And the opposite angles C_{ii} of matrix C is defined as follows:

$$C_{ii}(t) = -\sum_{\substack{j=1 \\ i\neq j}}^{N} C_{ij}(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} C_{ji}(t) = K_i(t) \quad (5)$$

Here, $|K_i|$ represents the degree of i th vertex at t moment.

Considering the complex network is dynamic, namely the network is changing over time, and the variation of network topological structure is random rather than fixed time or fixed times number. In the process of building network this paper firstly generates a network, such as scale-free network and small-world network, then achieve the links among network nodes by the following connection relation: if there exist a connection relation between ith vertex and jth vertex, they will connect each other with the larger probability at the next rebuilding network; otherwise, the connection probability will become small or zero. It may not be connected at the same time. Eventually, it is important to ensure the network without isolated vertices cluster. In the real life, the networks we have touched all are dynamic. For example: on the human social network, business trip or going abroad can lead to short disconnection; on the internetwork, the links among partial nodes can be broken by local network interruption; on the power control network, the damage of one node result in disconnection among nodes. Therefore, it is preferable to simulate the real network by building dynamic complex network.

$$C_{ij}(t) = \begin{cases} C_{ij}(t-1)^* p_1 & \text{there is a connection between } i \text{ and } j \\ C_{ij}(t-1)^* p_2 & \text{there is no connection between } i \text{ and } j \end{cases} \quad (6)$$

Here, $C_{ij}(t)$ represents network topology at t time on the complex network, $C_{ij}(t-1)$ represents network topology at $t-1$ time on the complex network, p_1 represents node i and j exist connecting relationship at $t-1$ time, namely the probability of node i and j connecting again at t time, p_2 represents node i and j don't exist connecting relationship at $t-1$ time, namely the probability of node i and j connecting again at t time.

Suppose there is no isolated vertices cluster on the dynamic network, so the topological matrix $C_{ij}(t)$ is an irreducible matrix. If all vertices have equal coupling strength, and the coupling strength between any two vertices is 1, namely $c = 1$, so the dynamic network system can be described as below:

$$\dot{X}_i = f(X_i) + \sum_{j=1}^{N} C_{ij}(t)\phi(t, X_j), \quad (i = 1, 2, 3, \ldots, N) \quad (7)$$

$$\dot{Y}_i = g(Y_i) + \sum_{j=1}^{N} C_{ij}(t)\phi(t, Y_j), \quad (i = 1, 2, 3, \ldots, N) \quad (8)$$

In the real life, it is difficult to maintain single and pure information source at the network nodes. For example, the network nodes always be interfered by other nodes or other information sources. In this paper, the priority is researching the hybrid synchronization of two heterogeneous chaotic systems on the complex dynamic and static networks, meanwhile, analyzing the differences of hybrid synchronization on the dynamic and static networks. Hence, the hybrid synchronization of complex dynamic network is worth to study and discuss.

Suppose the hybrid status of two heterogeneous chaotic systems can be denoted as $W = H(W)$. Here, this paper only considers the linear superposition of two heterogeneous hybrid chaotic systems, and the hybrid status can be described by:

$$W_i = H(W_i) = X_i + Y_i, \quad (i = 1, 2, 3, \ldots, N) \quad (9)$$

Here, $W = (w_1, w_2, w_3, \ldots w_N)^T \in \Re^n$ represent the state vector of hybrid systems. Because X_i and Y_i both are non-linear dynamic systems, the hybrid system W also is a new non-linear dynamic system.

This paper denotes time as a variable, which can describe the synchronization process of hybrid systems. $X_i(t, X_0)$ and $Y_i(t, Y_0)$ $(i = 1, 2, \ldots N)$ respectively represents the dynamic system X and Y. Here, $H(S(t))$ and $H(W_1(t)), H(W_2(t)), H(W_3(t)), \ldots H(W_N(t))$ respectively represents hybrid status. If formula (10) is workable, the nonautonomous complex dynamic network based on the description of Eq. (7) and (8) can be called as the synchronization of hybrid state $H(S(t))$ and $H(W_1(t)), H(W_2(t)), H(W_3(t)), \ldots H(W_N(t))$.

$$H(W_1(t)) \to H(W_2(t)) \to H(W_3(t)) \to, \ldots, \to H(W_N(t)) \to H(S(t)) \quad (10)$$

On the other words:

$$\lim_{t\to\infty} ||H(W_i(t)) - H(S(t))||_2 = 0, \quad (i = 1, 2, 3, \ldots, N) \quad (11)$$

Here, $S(t)$ is represented as an equilibrium point, a periodic orbit or an arbitrary chaotic attractor.