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The emergence of cooperation in tie strength models

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ABSTRACT

In this paper, we propose a tie strength model to explain the emergence of cooperation in spatial prisoner's dilemma games, assuming that cooperators preferentially allocate their investments to friends with strong ties. Two types of prisoner's dilemma models are examined in this study: the traditional two-strategy model considering only cooperators and defectors; the expanded three-strategy model consisting cooperators, defectors and extortioners. The results show that tie strength model contributes to the promotion of cooperation in both types of prisoner's dilemma games. However, we point out that the influence of the investment preference is quite different in the two prisoner's dilemma game settings. In the two-strategy prisoner's dilemma game, only small preference contributes to the promotion of cooperation. Once this preference exceeds a critical value, cooperation will be prohibited. We explain this phenomenon by arguing that extremely strong investment preference undermines the ability of cooperative clusters to resist defectors. Moreover, we extend the analysis into the three-strategy case and discover that the catalytic effect of extortioners can eliminate this first up and then down trend in the two-strategy model. The equilibrium fraction of cooperators is always positively correlated to the level of investment preference in three-strategy models.

1. Introduction

Cooperation is one of the foremost concerns of human being. It involves all aspects of human society, including physics, biology, sociology, economics, etc. [1,2]. Evolutionary game theory is the most widely adopted model to study cooperative behaviors. Despite in the classical game theory, Nash asserts that mutual defection is the only way to maintain an equilibrium in prisoner's dilemma game, many phenomena and experiments show that cooperation exists in reality [3–5]. Numerous hypotheses and models have been proposed to explain the emergence of cooperation, including coevolutionary rules [6–19], network reciprocities [20–22], etc.

Many relevant studies have found that the interactions among players and the resulting decisions are not purely random, but are affected by the social structures (networks) they reside in [23–26]. However, in most of these literatures, ties among different players are always considered to be identical. Only degree distributions of nodes are used to depict the spatial structures of players, where the diversity of tie strengths is ignored. In fact, the strength of ties is a key property to characterize social networks of players [27–31]. Interactions among players are not only influenced by the number of opponents they encounter, but also affected by the extent of intimacy between each pair of specific players [32,33]. For example, mothers are willing to sacrifice more to kids, close friends tend to provide altruistic support to each other. Cooperative behaviors are closely related to the extent of intimacy, to be more specific, tie strengths among players. Therefore, based on the above conjectures, we include diversified tie strengths into the analysis of spatial prisoner's dilemma games. We show that the proposed relational diversity can significantly promote cooperation, and this effect can be further enhanced if we consider the existence of extoritoners.

Moreover, in this paper, we not only consider the traditional prisoner's dilemma games with only two strategies (All C and All D), but also explore the effects of a newly discovered strategy called extortion strategy or zero determinant strategy. Extortion strategies are a subset of ZD strategies. Press and Dyson demonstrated that extortioners can impose a linear relationship between their own payoffs and the payoffs of the cooperators in donation games to ensure that an increase in an extortioner's payoff exceeds the increase in the corresponding cooperator's payoff by a fixed percentage controlled by parameter χ [34–36]. In the latter parts of this paper, the prisoner's dilemma game model we discussed consists of three possible strategies: Cooperation, Defection and Extortion [11,12]. We try to reveal how the tie strength affects the evolution of cooperation in PDG, and what will happen if we include extortioners into the analysis. The conclusion of this study

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Table 1.
Payoff matix of two strategy model.

	C	D
C	$b-c$	$-c$
D	b	0

Table 2.
Payoff matix of three strategy model.

	E_χ	C	D
E_χ	0	$\frac{(b^2-c^2)\chi}{b\chi+c}$	0
C	$\frac{b^2-c^2}{b\chi+c}$	$b-c$	$-c$
D	0	b	0

χ determines the surplus of the extortioner in relation to the surplus of the other player. Large χ value leads to strong extortion effect and extortioners become defectors if χ is positive infinite.

shows that tie strength and extortion jointly promote the emergence of cooperation in the spatial PDG settings.

In the classical evolutionary prisoner's dilemma game (PDG) model, each player has two feasible actions: cooperation (C) or defection (D). Both players get R (reward) for mutual cooperation and P (punishment) for mutual defection. A defector exploiting a cooperator gets T (the temptation to defect) and the exploited cooperator gets S (the sucker's payoff). R, P, T, S satisfy following conditions: $T > R > P > S$ and $2R > T+S$. To better illustrate the roles of diverse tie strengths in PDG, in the following we consider an important special case called the "donation game" (DG) In a DG, each player can cooperate by providing a benefit b to the other player at his or her cost c, with $0 < c < b$. Then, $T=b$, $R=b-c$, $P=0$, and $S=-c$. The payoff matrix of this two strategy model is illustrated in Table 1. To consider the three strategy model with extortioners, we follow the work of Hilbe et al., where extortion was studied in the realm of the donation game, the payoff matrix can be expressed as follows (see Table 2):

In order to explore the impact of tie strength, we randomly assign a tie strength value to each of the ties in the system. And these values will only be assigned once, before the game starts. Without loss of generality all tie strength values are between 0 and 1. The cooperators allocate their investments to friends proportional to the tie strengths among them. According to the common understanding, people tend to invest in their familiar friends, rather than those who called nodding friends. We use the parameter α to capture this behavioral preference. If $\alpha = 0$, a player has no preference on his friends, all friends get equal percentage of his investments if he decides to cooperate. If α tends to infinity, then the cooperator will put all his investments to his best friends and all other friends get nothing.

We find an interesting phenomenon that the introduction of tie strength has a significant impact on the emergence of cooperation. And this impact can be quite different subject to whether we include extortion strategy. In the absence of extortioners, the level of cooperation will be greatly enhanced by a small preference α, but will be significantly depressed if α exceeds a critical value. However, if extortion strategy is considered in the model, the threshold mentioned above disappeared. Cooperation can be improved continuously as α increases and the equilibrium level of cooperation is significantly enhanced.

The remainder of this paper is organized as follows: Section 2 gives a detailed description of our tie strength model under donation game framework by considering two situations: with and without extortioners. Simulation results are discussed in Section 3. In Section 4, we summarize the results and outline some important implications of our findings.

2. The model

2.1. The two strategy tie strength model

First, we consider a two strategy PDG with players located on an $L\times L$ square lattice with periodic conditions. Each player is allowed to interact with its four neighbors where self-interactions are excluded. In each round, player x is allowed to adopt the strategy of a randomly selected friend y with a probability $\mathrm{pr}_{x\to y}$ proportional to their payoff difference:

Classical donation game model assumes that the ability of a cooperator to invest in a network is proportional to his or her degree. A cooperator with k friends has a total amount of kc investments at the beginning of each round and will allocate the investments equally to all its k friends, i.e. each friend receives c and produces b out of this investment in each round. However, with the introduction of tie strength, we assume that a cooperator will preferentially allocate his investments to his good friends. The cooperator i will invest $I_c = k_i c \frac{ts_{ij}^\alpha}{\sum_n ts_{in}^\alpha}$ to his friend j. Therefore, the recipient j will get $\frac{b}{c}I_c$ from i's investment. If j is an extortioner, Here, ts_{ij} denotes the strength of the tie between the players i and j, and n runs over all i's friends. α is a tunable parameter controlling the investment preference of the cooperator. If $\alpha=0$, the model becomes a classical PDG.

The cooperator will equally distribute his investments and each of his friends gets a benefit b. If $\alpha > 0$, the cooperator will preferentially invest in his good friends. When $\alpha\to+\infty$, the cooperator will give all his investments to his best friend. The payoff of player x in the two strategy PDG can be expressed as:

$$P_{xC} = b\sum_{i\in\Gamma_{xC}} k_j \frac{ts_{ix}^\alpha}{\sum\limits_{j\in\Omega_{ii}} ts_{ij}^\alpha} - k_x c, \quad \text{if x is a cooperator} \tag{1}$$

$$P_{xD} = b\sum_{i\in\Gamma_{xC}} k_j \frac{ts_{ix}^\alpha}{\sum\limits_{j\in\Omega_{ii}} ts_{ij}^\alpha}, \quad \text{if x is a defector} \tag{2}$$

where Γ_{xC} denotes the set of x's friends adopting C and Ω_i represents the set of i's friends.

To introduce tie strength, we generate stochastic tie strength values ts_{ij} for each tie only once before the start of the simulation from the (0,1) interval. It is worth mentioning that we set $ts_{ij}=ts_{ji}$ for each relationship, assuming that the strength of a relationship is mutual and reciprocal.

2.2. The three strategy tie strength model

If we include extortion into the model, three strategies are available in the strategy space: cooperation (C), defection (D) and extortion (E). The cooperator i will invest $I_c = k_i c \frac{ts_{ij}^\alpha}{\sum_n ts_{in}^\alpha}$ to his friend j. Therefore, the recipient j will get $\frac{b}{c}I_c$ from i's investment if j is a cooperator or a defector. When a cooperator i encounters an extortioner j, then the cooperator i gets $\frac{(\frac{b}{c}I_c)^2 - I_c^2}{\frac{b}{c}I_c\chi + I_c} = \frac{b^2-c^2}{bc\chi+c}I_c$ and the extortioner gets $\frac{b^2-c^2}{bc\chi+c}I_c\chi$, respectively. Therefore, the payoff of player x in the three strategy prisoner's dilemma game can be expressed as:

$$P_{xC} = b\sum_{i\in\Gamma_{xC}} k_j \frac{ts_{ix}^\alpha}{\sum\limits_{j\in\Omega_i} ts_{ij}^\alpha} + \frac{b^2-c^2}{b\chi+c}\sum_{i\in\Gamma_{xE}} k_j \frac{ts_{ix}^\alpha}{\sum\limits_{j\in\Omega_i} ts_{ij}^\alpha} - k_x c,$$
$$\text{if x is a cooperator} \tag{3}$$