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Nonlinear dynamics and contact interactions of the structures composed of beam-beam and beam-closed cylindrical shell members

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ABSTRACT

Nonlinear beam-beam and beam-cylindrical shell contact interactions, where a beam is subjected to harmonic uniform load, are studied. First, the nonlinear dynamics governed by four nonlinear PDEs including a switch function controlling the contact pressure between the mentioned structural members are presented. Relations between dimensional and dimensionless quantities are derived, and the original problem of infinite dimension has been reduced to that of oscillator chains via the FDM (Finite Difference Method). Time histories, FFT (Fast Fourier Transform), phase portraits, Poincaré maps, and Morlet wavelets are applied to discover novel nonlinear chaotic and synchronization phenomena of the interacting structural members. Numerous bifurcations, full-phase synchronization of the beam-shell vibrations, the evolution of energy of the vibrating members, damped vibrations of the analyzed conservative system of the beam and the shell surface deformations for various time instants, as well as the buckling of the shell induced by impacts are illustrated and discussed, among others.

In addition, we have detected that in all studied cases, in spite of analyzing a large set of nonlinear ODEs approximating the behavior of interacting structural members, the scenario of transition from regular to chaotic dynamics follows the Ruelle-Takens-Newhouse scenario.

1. Introduction

There is a vast amount of literature devoted to modeling, analysis and design of beams, plates and shells. Even if the mentioned structural members are treated separately, there is a reach spectrum of their classification because of their geometry, shapes, types and design. In the case of beams, one may deal with simply supported and cantilever beams, straight and curved beams, slender (tapered) deep beams, Euler-Bernoulli beams, Timoshenko beams, Rayleigh-Timoshenko beams, layered sandwich beams, channel beams, cracked beams, etc.

In the case of plates, one may consider plates with different boundary conditions, thin and thick plates, moderately thick plates, curved and twisted plates, triangular/rectangular/circular plates, annular plates, rhombic/trapezoidal plates, layered/stiffened plates, sandwich/perforated/cellular plates, Kirchhoff/Mindlin/ Reissner-Mindlin plates, etc.

Since the vibrations of beams and plates have a long history in Classical Mechanics, we restrict ourselves here only to emphasize recent trends in the development of modeling and in analytical and numerical analysis, including optimization and control of the mentioned objects. Due to a large amount of the existing papers and books devoted to this subject, we are aimed here only at mentioning a few comprehensive examples of references associated with the introduced classifications, as it follows.

1. According to the industrial requirements, recently a wide interest in health monitoring of structural members, like beams, plates, and shells considered either as isolated or coupled objects, is observed. In particular, numerous approaches aimed at damage detection and using the already developed tools of nonlinear dynamical systems have been applied to identify existence and location as well as magnitude of the damages [1–4]. In this field of research, in spite of the direct numerical investigation of the governing PDEs, there are analytical approaches employing the so-called NNM (Nonlinear Normal Mode) methods, which is well-known in the theory of nonlinear dynamics [5]. On the other hand, since a crack locally changes flexibility

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of a structural member, it influences the values of the natural frequencies and mode shapes. Hence, it opens a way of measuring the location and possibly the size of a crack using vibration-based methods. In the latter case, the damages/cracks are modeled by simple mechanical massless components like a linear spring [6] or a rotational spring [7–9].

It should be noted that multi-span beams with single/multiple cracks have been investigated in references [10,11]. More recently, both identification and analysis of a single open crack modeled as an elastic rotational spring in a straight multi-span beam by means of the vibration-based nondestructive technique has been reported by Sharma et al. [12]. In the latter case, the classical Euler-Bernoulli theory has been employed to study the multi-span beam with and without crack.

2. The second class of research includes the multi-layer beams, plates, and shells made of composite materials widely applied in variety of industrial branches, civil and ship engineering, aeronautics, etc. Free as well as either externally or parametrically excited laminated beams, plates, and shells, having complex shapes and various arrangement of holes and cuttings occurred due to manufacturing/fabrication processes, are studied using a variety of numerical approaches (FEM, FDM, higher-order Bubnov-Galerkin methods, etc [13–19].).

   In overall, the mentioned laminated slender structural members are subjected to various kinds of boundary conditions, and the geometric, physical and design nonlinearity are taken into account while modeling and studying them.

3. The third class of the research is also generated by a high development of technology and mechanical and mechatronics engineering. Namely, fabrication of sensors and actuators employing microbeams and microplates sometimes requires novel modeling of beam/plates dynamics. However, usually the reduced-order models are applied.

   For instance, the classical Mathieu equation has been used for modeling the dynamics of a swing, the stability of ships and columns, and Faraday's surface wave patterns on the waters, while studying either a single-crystal silicon MEMS system [20] or a torsional silicon microresonator [21]. A nanomechanical system manufactured of three coupled beam resonators subject to magnetomotive driving has been modeled and analysed by Scheible et al. [22], where the Ruelle-Takens route to chaotic resonance has been reported.

4. Another challenging research topic is associated with appropriate and validated applications of numerous slender structural members theory to model the dynamics of thin-walled structures. There exist different widely described beam/plate/shell theories, including (i) Timoshenko, (ii) Euler-Bernoulli, (iii) Donnel-Mushtari, (iv) Love-Timoshenko, (v) Arnold-Warburton, (vi) Flugge-Byrne-Lurye, (vii) Reissner-Naghdi-Beny, (viii) Sanders, (ix) Vlasov, (x) Kemard-Simplified, and other [23,24].

   It strongly depends on a researcher, which theory should be employed to fit the real system behavior/experiments. In particular, sometimes novel theories should be proposed while modeling microbeams and microplates [25,26].

5. Another developing branch of research focuses on slender structures with contact boundary conditions. This research class includes problems of vibrations of beams, plates, and possibly shells with constrained dynamics. Two cantilever beams with motions bounded by fixed barriers have been studied by Emaci et al. [27]. Moon and Shaw have studied chaotic dynamics of a clamped beam, where the beam moved in two directions encountered a barrier in one of them [28]. Micro-vibro-impacts and frictional slips of the Euler-Bernoulli beams have been studied in reference [29], where a reduced order model has been employed.

6. The so far well-known theories and classical methods of solutions to the problems of vibrations of beams have been extended to study the so-called FG (Functionally Graded) beams, which are built of composite materials like ceramic-metal. Elasticity solution has been proposed by Sankar [30] who studied FG beams based on the Euler-Bernoulli theory. The layered FG beams have been studied analytically and experimentally by Kapuria et al. [31]. The first order shear deformation has been applied by Sina et al. [32] to solve the problem of free vibrations of FG beams. Kitipornchai et al. [33] analysed nonlinear vibrations of FG beams with edge cracks. More recently, the differential transformation method has been employed to analyze free vibrations of FG beams including a few types of elastic end constraints [34]. Fundamental and higher frequencies and the corresponding mode shapes have been reported.

   It should be also mentioned that the FG beams can work in high/low-temperature environments, and hence both thermo-mechanical loadings as well as temperature dependency of material properties should be taken into account (see [35] and references therein).

7. Beam/plates are 1D/2D continuous systems, i.e., systems with an infinite number of degrees of freedom, and hence their nonlinear multimodal behavior still attracts attention of numerous researchers focused on analysis of their global dynamics. Further extension of the modeling and analysis of the mentioned structural members includes, constrains to impact but, preserving, however, $N \to \infty$ degrees of freedom model. This way of thinking has been first employed by Wagg and Bishop [36], and extended by Wagg [37] (see also the references therein). The Galerkin truncation has been used matched with application of the nonsmooth modeling techniques to study nonlinear dynamics of a flexible impacting beam. The carried out numerical simulation results have been compared with the experimental results, putting emphasis on modeling one-sided impact phenomenon.

   There are also recently observed trends of energy harvesting from ambient vibrations, and in particular, attention is focused on harvesting vibrational energy at low frequencies and over large frequency intervals [38]. It includes an idea of the interaction between the driving beam and the generating beam and by driving a buckled beam including impact stops dissipating extra energy [39]. The carried out investigation has shown that one may increase electrical power generation, comparing with conventional approach, by controlling mechanical damping and the coupled vibrations. Lampart and Zapomel [40] studied attenuation of vibrations of the electromechanical system coupled with plate springs, taking into account nonlinear stiffness of springs, and an impact damping device. Most recently, the transverse impact of a beam hitting a fixed-fixed column employing the Timoshenko beam theory has been investigated by Khowitar et al. [41]. While analysing multiple impacts collision events have been divided into contact/separation phases .

8. It is highly required by industry to suppres vibrations of the structural members (here beams and plates) through control of their (usually) excessive vibrations, but treating them in a more natural way, i.e., taking into account their infinite-dimensional state space [42–44]. The mentioned topic is connected with an important theoretical challenge, as it has been reported in recent publications [45–47]. For example, He and Ge [48] have proposed the vibration control design for an Euler-Bernoulli beam with the boundary output constraint employing a novel barrier Lyapunov function and demonstrating the vibration suppressions.

   The so far carried out review of a state-of-the art of recent achievements and trends aimed at modeling, monitoring, simulation and control of vibro-impacting structural members, in-