Chaos, Solitons and Fractals 91 (2016) 695–701

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Abnormal cascading failure spreading on complex networks

Jianwei Wang[a], Enhui Sun[a], Bo Xu[a,*], Peng Li[b], Chengzhang Ni[a]

[a] *School of Business Administration, Northeastern University, Shenyang 110819, PR China*
[b] *Department of Mathematics, Zhongshan High School of Northeast, Shenyang 110819, PR China*

ARTICLE INFO

Article history:
Received 10 November 2015
Revised 22 March 2016
Accepted 21 August 2016

Keywords:
Cascading failure
Node weight
Betweenness
Robustness

ABSTRACT

Applying the mechanism of the preferential selection of the flow destination, we develop a new method to quantify the initial load on an edge, of which the flow is transported along the path with the shortest edge weight between two nodes. Considering the node weight, we propose a cascading model on the edge and investigate cascading dynamics induced by the removal of the edge with the largest load. We perform simulated attacks on four types of constructed networks and two actual networks and observe an interesting and counterintuitive phenomenon of the cascading spreading, i.e., gradually improving the capacity of nodes does not lead to the monotonous increase in the robustness of these networks against cascading failures. The non monotonous behavior of cascading dynamics is well explained by the analysis on a simple graph. We additionally study the effect of the parameter of the node weight on cascading dynamics and evaluate the network robustness by a new metric.

1. Introduction

Modern human societies are supported by various functional networks, such as power grids, the Internet, and traffic networks. The safety of these networks has been one of the classical research topics [1–9]. In particular, many researchers focus on the robustness of real networks against cascading failures [10–17]. Cascading failures are a sort of phenomena that a random failure or intentional attack on one or a few nodes triggers successive breakdowns and leads to serious damage to the whole network. Some typical real-world examples of cascading failures are the large-scale blackouts in some countries [10–15], e.g., the blackouts of America in 2003, Italy in 2003, London in 2003, and northern India in 2012. In addition, frequent traffic paralysis in some big cities and Internet collapse [16,17] are also caused by cascading failures. In order to mitigate and prevent various cascading-failure-induced disasters in the real world, many researchers investigated a number of important aspects of cascading failures, including the models for describing the cascade phenomena [18–24], the efficiency of random or targeted attacks [25–31], the cascade control and defense strategies [32–39], cascading failures in the multiplex networks [40–42], the percolation in the interdependent networks [43–52], and so on.

In many infrastructure networks, some sort of flow is often required to realize its functionality and at the same time the flow plays a role of a load in the network, such as traffic flow in the traffic network, electric current in a power grid, and data packet on the Internet [33]. Therefore, in previous studies on cascading failures, how to quantify the load on a node or an edge is the central issue. In earlier studies, the initial load on a node or an edge was generally given by the betweenness centrality of the node or the edge. Applying the betweenness centrality, the pioneering work by Motter et al. [53] discuss cascade-based attacks on complex networks and demonstrate that the initial removal of the highest degree (or highest load) node leads a large-scale cascade and scale-free networks are more fragile against cascading overload failures than homogeneous networks. Based on the strategy of the intentional removals of nodes and edges before the propagation of the cascade, Motter [54] propose a simple method to reduce the size of cascades of overload failures and show that the size of the cascade can be drastically reduced with the intentional removals of nodes having small load and/or edges having large excess of load. Defining the load on a node by the efficient paths, Crucitti et al. [55] propose a simple model for cascading failures and show how the breakdown of a single node is sufficient to collapse the entire system simply because of the dynamics of redistribution of flows on the network. Although the betweenness method can be widely applied to define the initial load on a node or an edge, it may be invalid for quantifying the flow of physical quantities in real networks. Since only one unit of the data packet between any two nodes is transported along with the shortest path, the betweenness method cannot thus approximate the traffic load in real networks, ignoring the differences among nodes, the weight of every edge, and the preferential characteristic of the destination

* Corresponding author.
E-mail address: wgdlut@163.com (J. Wang).

http://dx.doi.org/10.1016/j.chaos.2016.08.007
0960-0779/

selection of the flow. Therefore, we propose an improved betweenness measurement and construct a simple cascading model.

Considering the preferential mechanism of the flow transportation and the weights of nodes and edges, we analyze cascading dynamics on edges and develop a new method to assign the initial load of an edge. We propose a simple cascading model and study cascading failures on four artificial networks and two empirical networks. By disturbing the edge with the highest load, we observe an interesting and counterintuitive phenomenon in the cascading propagation. In these networks including the scale-free networks, the small-world network, the regular network, the random network, the traffic network, and the airport network, we find that, sometimes, improving the capacity of every edge inversely induces the robustness of these networks against cascading failures, evaluated by four metrics. We observe that the weights of nodes and edges have not affect the emergence of the abnormal cascading spreading in these networks. The non monotonous behavior of cascading dynamics is well explained by the analysis on a simple graph. Finally, we evaluate the network robustness by a new metric and give the correlation between the parameter of the node weight and the new metric.

2. The model

In previous studies, the initial load on a node or an edge is given by the betweenness centrality of the node or the edge, i.e., the initial load of a node or an edge is naturally defined to the total number of shortest paths passing through it. However, in previous cascading models [53–55] based on the betweenness method, network weights have not been taken into consideration, regardless of the facts that real networks display a large heterogeneity in the weights which have a strong correlation with the network topology. For example, in the airport network, the number of the traveling passengers on every airport may be different, and these passengers do not randomly select the destinations. Similarly, in the Internet, the data packets generated by every router are also different, and they select the destinations according to some rules. Motivated by this fact, we propose a new method to define the initial load of an edge.

Firstly, considering the heterogeneity in the capacity of nodes, we define the weight (or strength) of a node according to its local characteristics. Inspired by Refs. [56–60], we assume the weight w_i of node i to k_i^{α}, where α is a tunable weight parameter, governing the strength of the node weight, and k_i is the degree of node i. This assumption is supported by empirical evidence of real weighted networks [58,59], i.e., the bigger the node degree, the higher the node weight. Moreover, the ref [60] use this method to define the load or weight of a node. Based on the node weight, we study the preferential mechanism of the destination selection of flows. We use $F_{i\rightarrow}$ to denote the flow generated by node i. For simplicity, we assume $F_{i\rightarrow} = w_i$. We use $F_{i\rightarrow j}$ to denote the flow transported from node i to node j. In $F_{i\rightarrow}$, we assume that the flow transported from node i to node j ($i \neq j$) is proportional to the weight of node j, i.e.,

$$F_{i\rightarrow j} = F_{i\rightarrow} \frac{w_j}{\sum_{m\in N} w_m - w_i}, \tag{1}$$

where N is the set of nodes in a network.

Since the flow via the edge plays a role of a load, we focus on the effect of the flow transported between two nodes on the edges. We assume that the flow is transmitted along the shortest paths with the edge weight connecting two nodes. Without loss of generality, in later simulations, the weight of every edge is assigned by random numbers of the uniform distribution. If there is more than one shortest path with the edge weight connecting two given nodes, the flow is divided evenly at each branching point (see Fig. 1).

The initial load of an edge is defined as the amount of the flow between pairs of nodes that run through that edge. In Fig. 2, by the total amount of the flow passing through a given edge, we calculate the initial load L_{ij} of edge ij, i.e.,

$$L_{ij} = \sum_{m\in N, n\in N} L_{ij}^{(m,n)}, \tag{2}$$

where $L_{ij}^{(m,n)}$ denotes the load via edge ij among the flow transported between the ordered node pair m and n. In particular, when $\alpha = 0$ and the weight of every edge is same, our method to define the initial load is as same as one of the betweenness centrality [53–55]. Following Refs. [13–15,18,19,26,27,53–55], we assume the capacity C_{ij} of edge ij to be proportional to its initial load L_{ij}.

$$C_{ij} = (1+\beta)L_{ij}, \tag{3}$$

where the parameter $\beta \geq 0$ is the tolerance parameter. This is a realistic assumption in real networks, since the capacity cannot be infinitely large because it is limited by the cost. With such a definition of capacity, initially the network is in a stationary state in which the load at each edge is not bigger than its capacity. The removal of an edge, changing the topological structure and the path lengths among some nodes, destroy the balance of the load and lead to a global redistribution of loads in the network. After updating the load of every remaining edge, some edges of overloads will be removed from the network, since their limited capacities are insufficient to handle the extra load. This leads to a new redistribution of loads and triggers a cascade of overload failures. This cascading process stops only when the capacity of every remaining edge is not smaller than its updated load.

After the cascading propagation is over, we calculate the number G of nodes in the largest connected component, the number S_E of failed edges, and the avalanche size S_N of failed nodes. We additionally propose a new metric S_C, i.e., the number of the connected component (there are at least two nodes in every connected component). S_C can quantify the degree of fragmentation of the whole network. In later simulations, we use G, S_E, S_N, and S_C to evaluate the network robustness against cascading failures.

3. The analysis of the cascading model

Since the network structure plays an important role in the cascading propagation, we first select four types of classical artificial networks to study cascading dynamics on them. These networks are the BA scale-free networks [61], the Ring networks, the WS small-world networks [62], and the ER random networks. BA networks can be constructed as follows: starting from m_0 fully connected nodes, a new node with $m (m \leq m_0)$ edges is added to the existing network at each time step according to the preferential attachment, i.e., the probability of being connected to the existing node i is proportional to its k_i. We set $N = 200$ and $m_0 = 2$, $m = 2$, i.e., the average degree $\langle k \rangle$ is about 4. In the Ring network, the starting point is a N nodes ring, in which each node is symmetrically connected to its $2m$ nearest neighbors for a total of $m \times N$ edges. Here, we also set $N = 200$ and $m = 2$, i.e., the average degree $\langle k \rangle$ is about 4. WS networks can be constructed as follows: starting from a ring network with 200 nodes and 4 edges per node, we rewire each edge at random with the probability p. When $p = 0.01$, WS networks have both the small-world property and a high clustering coefficient. Therefore, in simulations, we set $p = 0.01$ in WS networks. To compare the effect of the network structure on cascading dynamics, we generate ER network with 200 nodes and $\langle k \rangle = 4$. We propose a new method as follows: starting from a ring network with 200 nodes and 4 edges per node, we rewire each edge at random with probability $p = 1$.