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Sector-condition-based results for adaptive control and synchronization of chaotic systems under input saturation

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ABSTRACT

This paper addresses the design of adaptive feedback controllers for two problems (namely, stabilization and synchronization) of chaotic systems with unknown parameters by considering input saturation constraints. A novel generalized sector condition is developed to deal with the saturation nonlinearities for synthesizing the nonlinear and the adaptive controllers for the stabilization and synchronization control objectives. By application of the proposed sector condition and rigorous regional stability analysis, control and adaptation laws are formulated to guarantee local stabilization of a nonlinear system under actuator saturation. Further, simple control and adaptation laws are developed to synchronize two chaotic systems under uncertain parameters and input saturation nonlinearity. Numerical simulation results for Rössler and FitzHugh–Nagumo models are provided to demonstrate the effectiveness of the proposed adaptive stabilization and synchronization control methodologies.

1. Introduction

The stabilization and synchronization problems of chaotic systems have received a great deal of attention among researchers in many fields of science and engineering due to their potential applications, for instance robotics, signal processing, biomedicine, secure communication, power systems, image processing, aerospace, mechanics, physics, and chemical reactions (see, [1–12]). Stabilization is a fundamental objective of any control system that can be achieved by application of a nonlinear or an adaptive control theory [1–5]. Synchronization of different nonlinear systems is an unsolved dilemma in control; the solution to which can be applied to resolve trajectory tracking and chaos synchronization problems. Trajectory tracking control has vast applications in robotics, ship steering, and satellite tracking [6,7], while chaos synchronization has a very significant impact on secure communication, aerospace engineering, and medical therapies [8–16]. Incorporation of actuator saturation to the formulation of nonlinear and adaptive algorithms for the stabilization and synchronization of dynamical systems against windup effects is an appealing research topic owing to the physical constraints on actuators and inaccurate measurement of system parameters.

Adaptive controls of chaotic systems against input saturation and unknown parameters, by employing global control schemes and ensuring bounded control errors, have been explored in the recent works [17–21]. An adaptive tracking

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control approach of uncertain nonlinear systems in the presence of non-symmetric input nonlinearity was developed in [17] by exploiting the spectral radius of the control coefficient matrix. In [18], neural network based control scheme was employed for stochastic single-input single-output nonlinear systems with bounded control signal constraints. The work in [19] addressed a constrained control problem by application of a neutral observer to compensate the undesirable effects for the control of nonlinear time-delay systems. Robust adaptive neural network based control strategies for nonlinear multi-input multi-output systems under nonlinear inputs based on adaptive radial basis function and variable structure control theory were proposed in [20,21]. The problem of synchronization of chaotic systems under input saturation was addressed in a recent work [13] by ensuring a region of stability in terms of the difference between states of the nonlinear oscillators. Sector-condition-based control solutions [22–26] for controlling linear and nonlinear systems with known parameters under input saturation, suitable for multi-objective synthesis and ensuring windup protection, global stability, local stability with a large region of convergence, robustness and disturbance rejection, are abundant in the literature. However, the applicability of the traditional sector conditions to the adaptive control problems or derivation of new sector conditions appropriate for the adaptive systems remains uncertain up to this date. Consequently, investigation of a new sector condition, relevant to the nonlinear and, particularly, to the adaptive control problems, is necessary in order to open up future research directions and possibilities. To the best of our knowledge, local adaptive control strategies for stabilization and synchronization of chaotic systems under input saturation, guaranteeing asymptotic stability by employing a sector condition for the input saturation nonlinearity, are deficient in the literature.

This paper discusses sector-condition-based adaptive stabilization and synchronization of chaotic systems with unknown parameters under the input saturation constraint. A novel sector condition for a saturation (or dead-zone) function is developed, which is more general than the traditional global and local sector conditions (e.g. [13,22–26]). In addition, a local adaptive scheme based on linear matrix inequalities (LMIs) is established, which exploits the proposed sector condition. This local stabilization methodology guarantees a region of convergence and offers a tolerable bound on the adaptive and nonlinear components in the control law, both of which can be enlarged to fulfill the desired design requirements. In contrast to the conventional stabilization methods like [17–21], the present adaptive stabilization approach is computationally simple due to the incorporation of the proposed sector condition and owing to the relaxation of additional adaptive terms for compensation of the saturation function. Adaptive compensation of the saturation or the dead-zone function that can lead to the inferior closed-loop performance is not employed in our approach. The proposed control approach guarantees convergence of the state vector to the origin in contrast to the traditional methods based on the uniformly ultimately bounded stability criteria. Moreover, a novel adaptive control strategy for synchronization of different chaotic systems facing actuator saturation, which can be readily applied to the trajectory tracking as well as to the chaos synchronization problems, is derived herein. In contrast to the existing method in [13], the present synchronization control approach is capable of dealing with the uncertain parameters by virtue of the proposed constrained adaptation laws. It is demonstrated that the conventional adaptive control approaches (like [11]), irrespective of the input saturation, can be derived as specific cases of the proposed adaptive treatment. Numerical simulation results of the proposed adaptive control schemes are provided for the stabilization and the synchronization of chaotic Rössler systems and FitzHugh–Nagumo neurons.

This paper is organized as follows. Section 2 presents the problem description. Section 3 details the theoretical conditions for the new sector condition and the proposed adaptive approaches for the control and synchronization of chaotic systems. Section 4 analyzes simulation results. Section 5 draws conclusions. Standard notation is used throughout the paper. The notation $\|\cdot\|$ symbolizes the Euclidian norm of a vector. A symmetric positive (or semi-positive) definite matrix X is referred to as $X > 0$ (or $X \geq 0$). $Z_{(i)}$ denotes the ith row of a matrix Z. The conventional saturation function, for a control signal $u \in R^m$, is given by $\mathrm{sat}(u) = \mathrm{sign}(u_{(i)}) \min(\bar{u}_{(i)}, |u_{(i)}|)$, where $\bar{u}_{(i)} > 0$ denotes the ith bound on saturation.

2. Problem description

Consider a nonlinear system with unknown parameters given by

$$\frac{dx}{dt} = f(x) + F(x)\alpha + Ax, \tag{1}$$

where $x \in R^n$ is the state vector. Functions $f(x) \in R^n$ and $F(x) \in R^{n \times m}$ represent the nonlinear components, and matrix $A \in R^{n \times n}$ denotes the linear component of the system (1) dynamics. The unknown parameters are represented by the vector $\alpha \in R^m$. Consider another nonlinear system with uncertain parameters under input saturation, given by

$$\frac{dz}{dt} = g(z) + G(z)\beta + Az + \mathrm{sat}(u), \tag{2}$$

where $z \in R^n$, $u_{\mathrm{sat}} = \mathrm{sat}(u) \in R^n$, and $u \in R^n$ are the state, saturated control signal and control input vectors, respectively. The nonlinear dynamics of system (2) are symbolized by functions $g(z) \in R^n$ and $G(z) \in R^{n \times p}$. The vector $\beta \in R^p$ comprises the unknown parameters.

Assumption 1. *The unknown parameters in systems (1)–(2) are bounded by* $\|\alpha - \alpha_o\| \leq \alpha_{\max}$ *and* $\|\beta - \beta_o\| \leq \beta_{\max}$, *where* α_o *and* β_o *are the expected values of* α *and* β, *respectively.*

For simplicity, α_o and β_o can be selected as zero. If prior information on α and β is known, it also can be incorporated. The aims of the present study are to address the following three problems:

Problem 1. Devise a generalized sector condition for the saturation function, sat(u), which can be employed to design a nonlinear and/or adaptive controller for a nonlinear system such as (2).

Problem 2. Derive a nonlinear adaptive control law to ensure the local stability within a large region of convergence for system (2) under input saturation satisfying $\|\beta - \beta_o\| \leq \beta_{\max}$.