Chaos, Solitons and Fractals 77 (2015) 230–234

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Mutual punishment promotes cooperation in the spatial public goods game

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ARTICLE INFO

Article history:
Received 3 February 2015
Accepted 3 June 2015
Available online 22 June 2015

PACS:
02.50.Le
87.23.Kg
87.23.Ge

Keywords:
Cooperation
Public goods game
Punishment

ABSTRACT

Punishment has been proved to be an effective mechanism to sustain cooperation among selfish individuals. In previous studies, punishment is unidirectional: an individual i can punish j but j cannot punish i. In this paper, we propose a mechanism of mutual punishment, in which the two individuals will punish each other if their strategies are different. Because of the symmetry in imposing the punishment, one might expect intuitively the strategy to have little effect on cooperation. Surprisingly, we find that the mutual punishment can promote cooperation in the spatial public goods game. Other pertinent quantities such as the time evolution of cooperator density and the spatial distribution of cooperators and defectors are also investigated.

1. Introduction

Cooperation is widely existent in human society and animal world [1]. Understanding and searching for mechanisms that can generate and sustain cooperation among selfish individuals remains to be an interesting problem. Evolutionary game theory represents a powerful mathematical framework to address this problem [2]. Various game models have been introduced, among which the public goods game (PGG) has been a prevailing paradigm [3]. Due to the rapid development of complex networks [4], the PGG and other evolutionary game models have been extensively studied in various kinds of structured populations [5,6], including regular lattices [7–16], small-world networks [17], scale-free networks [18–24], dynamic networks [25–28], and interdependent networks [29].

Both theoretical [30–38] and experimental [39–44] studies have shown that punishment is an effective way to enforce cooperative behavior in spatial evolutionary games. Traditionally, punisher are cooperators or, alternatively, of defectors [34,38]. Those that are punished bear a fine while the punishers usually bear a cost of punishment [45]. In previous studies, objects of punishment are individuals who hold a specific strategy (usually is deemed to be defection). However, the punished strategy may not be fixed but depends on the surrounding environment, e.g., on neighbors' strategies. Psychological experiments have demonstrated that, an individual tends to coincide with others in behavior or opinion [46]. There is a psychological or financial punishment of dissent, with humans trying to attain social conformity modulated by peer pressure [46–48].

Based on the above consideration, we propose a mechanism of punishment in which an individual will punish neighbors (no matter cooperators or defectors) who hold

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http://dx.doi.org/10.1016/j.chaos.2015.06.002
0960-0779/

the opposite strategy. Unlike in previous models where additional strategies of punishment are introduced, there are only two strategies (pure cooperators and pure defectors) in our model. The punishment is mutual in our model, that is, an individual i who punishes individual j is also punished by j. Thus, the cost of punishment can be absorbed into the punishment fine. Because of this symmetry at the individual or "microscopic" level, intuitively one may expect the punishment not to have any effect on cooperation. Surprisingly, we find that symmetric punishment can lead to enhancement of cooperation in the spatial PGG.

2. Model

Our model is described as follows.

Players are located on a $L \times L$ square lattice with periodic boundary conditions. Every player occupies a lattice point and has four neighboring points. Each player i participates in five PGG groups sponsored by i and its four neighbors respectively. A PGG group is composed of a sponsor and its four neighbors. Thus the size of each PGG group is five.

At each time step, every cooperator (denoted by C) contributes a total unit cost shared equally by five involved PGG groups. Defectors (denoted by D) invest nothing. The total cost of a group is multiplied by a factor, and is then redistributed uniformly to all the five players in this group. We denote i's strategy as $s_i = 1$ for cooperation and $s_i = 0$ for defection. The payoff that player i gains from the group sponsored by player j is

$$p_{i,j} = -\frac{s_i}{5} - \alpha|s_i - s_j| + \frac{r}{5}\sum_{x=0}^{5} s_x, \tag{1}$$

where $x = 0$ stands for player j, $x > 0$ represent the neighbors of j, r is the multiplication factor, and α is the punishment fine. The total payoff of the player i is calculated by

$$P_i = \sum_{j \epsilon \Omega_i} p_{i,j}, \tag{2}$$

where Ω_i denotes the community of neighbors of i and itself.

Initially, cooperators and defectors are randomly distributed with the equal probability 0.5. After each time step, all individuals synchronously update their strategies as follows. Each individual i randomly chooses a neighbor j and adopts the neighbor j's strategy with the probability [49]:

$$W(s_i \leftarrow s_j) = \frac{1}{1 + \exp[(P_i - P_j)/K]}, \tag{3}$$

where K characterizes the noise introduced to permit irrational choices.

3. Results

We assume that players occupy nodes on a 100×100 square lattice and the noise $K = 0.1$. The key quantity for characterizing the cooperative behavior of the system is the fraction of cooperators ρ_c in the steady state. In all simulations below, ρ_c is obtained by averaging over the last 2000 time steps of the entire 30,000 time steps. Each data are obtained by averaging over 200 different realizations.

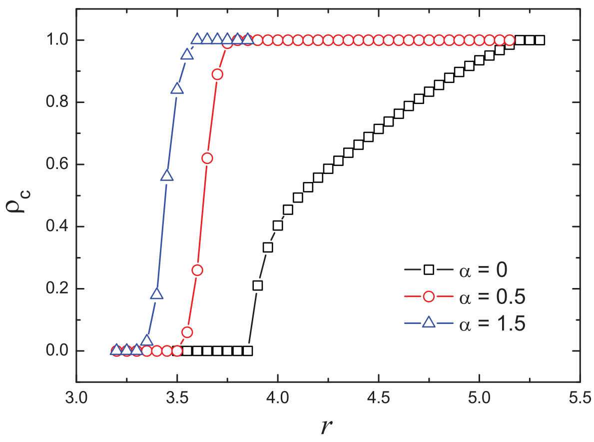

Fig. 1. The fraction of cooperators ρ_c as a function of the multiplication factor r for different values of the punishment fine α. For each value of α, ρ_c increases with r.

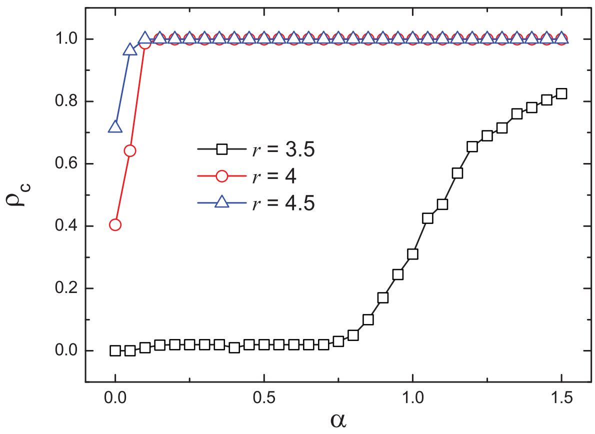

Fig. 2. The fraction of cooperators ρ_c as a function of the punishment fine α for different values of the multiplication factor r. For each value of r, ρ_c increases with α.

Fig. 1 shows the fraction of cooperators ρ_c as a function of the multiplication factor r for different values of the punishment fine α. From Fig. 1, we can see that for any given value of α, ρ_c increases from 0 to 1 as r increases. Fig. 2 shows the fraction of cooperators ρ_c as a function of the punishment fine α for different values of the multiplication factor r. From Fig. 2, one can find that, for a fixed value of r, ρ_c increases with α.

Next, we examine the time evolution of cooperator density $\rho_c(t)$ for different values of the punishment fine α when the multiplication factor $r = 4$. From Fig. 3, one can see that for each value of α, $\rho_c(t)$ decreases at the beginning and then increases. For $\alpha = 0$, the lowest value of $\rho_c(t)$ is about 0.06 and $\rho_c(t)$ will reach a steady value (about 0.4). For $\alpha = 1$, the lowest value of $\rho_c(t)$ is about 0.3, which is much larger than that for $\alpha = 0$. Besides, for $\alpha = 1$, cooperators will occupy the whole system in the end.

It has been known that the formation of clusters plays an important role in maintaining cooperation in spatial games [50–52]. A cooperator (defector) cluster is defined as a connected component (subgraph) fully occupied by cooperators (defectors). Within clusters, cooperators can assist each