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Effects of stochastic resonance for linear–quadratic detector

Shujun Liu*, Ting Yang, Xinzheng Zhang

College of Communication Engineering, Chongqing University, Chongqing 400044, China

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ABSTRACT

In this paper, the stochastic resonance (SR) effects of the fixed linear–quadratic (L–Q) detector in binary hypothesis testing problems is investigated. The effects include the improvement of the overall performance of the detector wherein the output signal-to-noise ratio (SNR) of receiver and the detection probability (P_D) are increased and the false-alarm probability (P_{FA}) is reduced at the same time. The SNR of a noise modified detector is derived. Since the improvement of the SNR does not mean the improvement of detection performance, the noise enhanced detection performance in terms of P_{FA} and P_D are discussed graphically and elaborated through an example of Gaussian background noise. Furthermore, the conditions are deduced to determine whether the overall performance of the detector can be improved or not. In addition, the form of the suitable noise probability distribution function (PDF) is determined and the PDF of noise that satisfies the conditions to improve the overall performance of detector are derived. Finally, an illustrative example is presented to verify the theoretical results.

1. Introduction

Linear–quadratic (L–Q) detector are widely used in signal processing problem. In general, the L–Q detector includes a L–Q receiver and a decision threshold. The L–Q receiver usually consists of a linear term and a quadratic term. When the coefficient of the quadratic term is zero, the L–Q detector represents the linear detector. Especially, the L–Q detector can also be equivalent to the sign detector if the threshold equals zero. Otherwise, if the coefficient of the quadratic term is nonzero, the L–Q detector represents a quadratic detector. As a result, the L–Q detector has received considerable attentions for many years. The optimal L–Q system for detection was proposed long ago by adjusting receiver parameters to meet some criteria. The complete solution of optimal L–Q systems for detection is presented in [1] by using finite fourth-order moments for an arbitrary probability distribution. However, in most cases, the L–Q detector is fixed and less efficient than the optimal detector. As a consequence, how to enhance the performance of the suboptimal L–Q detector without changing its structure becomes a significant topic.

With the preceding as motivation, the effect of adding noise to detector has been widely investigated [2–20]. Noise is usually filtered out by various signal processing algorithms and/or filters. Nevertheless, as a counterintuitive phenomenon, the useful role of noise has been observed and applied in variety of quantizers [2–6] and detectors [7–19] since the concept of stochastic resonance (SR) first proposed by Benzi in 1981 [20]. In [14], the improvement of asymptotic efficiency with constant false-alarm rate (CFAR) by adding noise is analyzed under the assumption that the probability density function (PDF) of the observed data and the background noise are symmetric. Among several criteria, SNR becomes a key factor in many cases and using SR to gain the high SNR is a classic topic of interest [21–30]. Up to data, though many methods have been presented to improve the SNR, the problem how to get the larger SNR and the improved detection performance simultaneously is little considered.

* Corresponding author. Tel.: +86 13436044455; fax: +86 02365103544.
E-mail address: liusj03@126.com, liusj@cqu.edu.cn (S. Liu).

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In this paper, the effect of adding noise to the L–Q detector for a binary hypothesis-testing problem is investigated. We not only concern the output SNR of receiver but also pay close attention to the detection performance by adding additive noise. However, it is not easy to find the optimal additive noise directly to improve the SNR and the detection probability (P_D) while reduce the false-alarm probability (P_{FA}). We first find the suitable noise to increase the SNR under the conditions that the detector is fixed and the distribution form of background noise is assumed unknown except that certain properties such as the moments are given. It means that we can only add suitable noise without adjusting the parameters and/or the thresholds of the detector to improve the performance of detector. In this case, in order to guarantee the additive noise can improve the output SNR or the deflection of the L–Q receiver, we derive the conditions that the moments of the additive noise have to meet. As the improvement of SNR does not mean the improvement of detection performance at the same time, the added noise need more constrains to realize the improvement of detection performance. Although the deflection can be derived with inadequate prior information, in order to get the optimal noise to enhance detection performance we need to know the distribution of background noise, as shown in [11]. In many practical matters, as time goes by, more information related to the PDF of background noise could be available. Thus, we derive the exact form of the suitable noise PDF and present a simple way to determine the range scope from which the additive noise can be chosen. Further, in Gaussian noise background, the optimal noise PDFs for different parameters of receiver are given by the graphical method, where two special cases are realized as the minimization of P_{FA} without decreasing P_D and the maximization of P_D without increasing P_{FA}, respectively. Besides, the additive noise which can increase the detection probability and decrease the false-alarm probability simultaneously is derived. Finally, the conditions for the improvement of P_D, output SNR and P_{FA} at the same time are discussed from multiple perspectives. The main contributions of this paper can be summarized as below:

- Determination of SR noise to improve the output SNR of the L–Q receiver.
- Derivation of SR noise which can increase P_D and decrease P_{FA} at the same time.
- Analysis of the noise enhanced output SNR without deteriorating the detectability.

The remainder of this paper is organized as follows. In Section 2, the model of output SNR, or the deflection criterion of a L–Q detector is given and the optimal noise that can enhance deflection and the conditions the additive noise need to meet are derived in different situations. In Section 3, the false-alarm and detection probabilities of L–Q GCD are analyzed and the value scope of additive noise which can improve the detection performance are derived by a graphical method. In Section 4, the conditions of additive noise which can enhance deflection, false-alarm and detection performance simultaneously are discussed. Finally, an example and some simulations are presented in Section 5 and conclusions are made in Section 6.

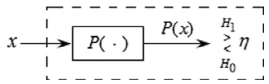

Fig. 1. The original detector.

2. Noise enhanced L–Q detector based on deflection criterion

2.1. Problem formulation

Consider a binary hypothesis-testing problem as follows:

$$\begin{cases} H_0 : x = n \\ H_1 : x = n + s \end{cases}, \tag{1}$$

where x is an observation, n is the background noise, s is a known constant and $s > 0$ which means the direct current (DC) signal. Suppose that the moments of background noise $m_k = E(n^k)$ are known for $1 \le k \le 4$ and the first moment $m_1 = E(n) = 0$, namely, the mean value of background noise is zero.

Let $P(x)$ be the output of a receiver in an arbitrary L–Q detector [21], the corresponding output SNR, or deflection is defined by

$$D(P) = \frac{[E_1(P) - E_0(P)]^2}{V_0(P)}, \tag{2}$$

where x is an observation vector, E_1 and E_0 represent the expectations of $P(x)$ under H_1 and H_0, respectively. V_0 means the variance under H_0 and the decision process of the detector is shown in Fig. 1.

Suppose the receiver can be written as $P(x) = \boldsymbol{h}^T\boldsymbol{x}$, the PDF of the background noise n is unknown and the mean value of x under H_1 and H_0 are known, which are denoted by $\boldsymbol{e_1}$ and $\boldsymbol{e}_0$, respectively. The variance of x under H_0 is $\boldsymbol{V}_0$. Then the expectations of $P(x)$ under H_1 and H_0 can be obtained as $E_1 = \boldsymbol{h}^T\boldsymbol{e}_1$ and $E_0 = \boldsymbol{h}^T\boldsymbol{e}_0$. The variance of $P(x)$ under H_0 can be expressed as $\boldsymbol{h}^T\boldsymbol{V}_0\boldsymbol{h}$. With these notations, the deflection of $P(x)$ can be rewritten

$$D(P) = \frac{[\boldsymbol{h}^T(\boldsymbol{e}_1 - \boldsymbol{e}_0)]^2}{\boldsymbol{h}^T\boldsymbol{V}_0\boldsymbol{h}}. \tag{3}$$

If $\boldsymbol{V}_0$ has no zero eigenvalue, $D(P)$ is a limited value and

$$\begin{aligned} &[\boldsymbol{h}^T(\boldsymbol{e}_1 - \boldsymbol{e}_0)]^2 \\ &= [\boldsymbol{h}^T\boldsymbol{V}_0\boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0)]^2. \\ &\le \left\langle \boldsymbol{h}^T\boldsymbol{V}_0, \boldsymbol{h}^T\boldsymbol{V}_0 \right\rangle \cdot \left\langle \boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0), \boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0) \right\rangle \\ &= \boldsymbol{h}^T\boldsymbol{V}_0(\boldsymbol{h}^T\boldsymbol{V}_0)^T(\boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0))^T\boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0) \\ &= (\boldsymbol{h}^T\boldsymbol{V}_0\boldsymbol{h})[(\boldsymbol{e}_1 - \boldsymbol{e}_0)^T\boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0)]. \end{aligned} \tag{4}$$

from the Schwarz inequality [3], which shows that the maximum value of $D(P)$ is obtained as $(\boldsymbol{e}_1 - \boldsymbol{e}_0)^T\boldsymbol{V}_0^{-1}(\boldsymbol{e}_1 - \boldsymbol{e}_0)$ if $\boldsymbol{V}_0$ is a solution of equation $\boldsymbol{V}_0\boldsymbol{h} = \boldsymbol{e}_1 - \boldsymbol{e}_0$ when detector is fixed. In this paper, the output of the L–Q receiver $P(x)$ is shown as

$$P(x) = ax + b(x^2 - m_2) = [a, b][x, x^2 - m_2]^T, \tag{5}$$