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Effective porothermoelastic properties of transversely isotropic rock-like composites

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Abstract

The present work is devoted to the determination of the overall porothermoelastic properties of transversely isotropic rock-like composites with transversely isotropic matrix and randomly oriented ellipsoidal inhomogeneities and/or pores. By using the solution of a single ellipsoidal inhomogeneity arbitrarily oriented in a transversely isotropic matrix presented by Giraud et al. [A. Giraud, Q.V. Huynh, D. Hoxha, D. Kondo, Effective poroelastic properties of transversely isotropic rocks-like composites with arbitrarily oriented ellipsoidal inclusions, Mechanics of Materials 39 (11) (2007) 1006–1024], it is possible to observe the effect of the shape and orientation distribution of inhomogeneities on the effective porothermoelastic properties. Based on recent works on porous rock-like composites such as shales or argillites, an application of the developed solution to a two-level microporomechanics model is presented. The microporosity is homogenized at the first level, and multiple solid mineral phase inclusions are added at the second level. The overall porothermoelastic coefficients are estimated in the particular context of heterogeneous solid matrix. The present model generalizes to transversely isotropic media a recently developed two-level model in the simpler case of isotropic media (see Giraud et al. [A. Giraud, D. Hoxha, D.P. Do, V. Magnenet, Effect of pore shape on effective thermoporoelastic properties of isotropic rocks, International Journal of Solids and Structures 45 (2008) 1–23]). Numerical results are presented for data representative of transversely isotropic rock-like composites.

Keywords: Eshelby tensor; Transverse isotropy; Poroelasticity; Porothermoelasticity; Ellipsoidal inclusion; Micromechanics; Biot tensor; Thermal expansion coefficient

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1. Introduction

This study is devoted to the overall porothermoelastic response of porous rock-like composites composed of a transversely isotropic matrix with embedded pores and solid inhomogenities. The homogenization method is based upon the Eshelby tensor approach. The shape of pores and solid inhomogeneities are respectively supposed ellipsoidal and spherical. This assumption corresponds to sedimentary rock-like composites

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such as shales and argillites (see also [31,10,11]). The fundament of the homogenization method is the Eshelby and Hill polarization tensors of an isolated ellipsoidal inhomogeneity embedded in an infinite transversely isotropic matrix. The solution of this problem is well known in the particular case of an isotropic elastic matrix. An extensive bibliography on homogenization methods can be found in [36] for the general case of composites of the matrix-inclusion type, in [19] for the particular case of transversely isotropic rocks and in [34,14] for anisotropic rocks. See also among others [1,2]. In these works the well known Eshelby $\mathbb{S}^{E}$ and Hill tensors $\mathbb{P}$ for ellipsoidal inclusions in an isotropic medium [8,9,21,22] have been used through homogenization schemes. The solution of the elastic inhomogeneity problem is analytical when the homogeneity is aligned with the directions of the transversely isotropic matrix. A general solution of this problem has been presented by [33,29] for various inhomogeneities (including ellipsoidal and spherical inhomogeneity) in a transversely isotropic material (see also [12]). Recently, a semi-explicit solution has been presented in [13] for the problem of an arbitrarily oriented ellipsoidal inhomogeneity in a transversely isotropic matrix. Based on a numerical integration of the exact Green function provided by [23] for the transversely isotropic media, the Hill tensor $\mathbb{P}$ is obtained for an arbitrarily oriented ellipsoidal inclusion. The problem of the arbitrarily oriented ellipsoidal inhomogeneity is fundamental and allows to separate respective effects of matrix anisotropy and of inhomogeneities' orientation distribution and shape on the overall anisotropy. In this paper, the solution developed in [13] will be use to take into account different orientation distribution of the pores: aligned, random and preferentially oriented.

For low and *moderate* volume fraction of inclusions and for materials such as deep clayey formations, the Mori–Tanaka homogenization scheme provides correct results for elastic and poroelastic properties (see [31]) and it will be used in this paper. It may be noticed that in order to account for the spatial distribution of non spherical inclusions, the Ponte Castañeda–Willis scheme [24] can be used instead of Mori–Tanaka scheme (see [14] for applications of this estimate to porous rocks). Levin and Alvarez-Tostado [18] derived explicit constants for an inhomogeneous porothermoelastic medium, with application on rock-like materials, using the effective field method (EFM) as a particular case of self-consistent homogenization scheme.

In this paper a two level homogenization model is developed to the determination of the overall porothermoelastic properties of transversely isotropic rock-like composites. A recent presentation of this model in the particular case of isotropic porous rock-like composite has been done in [11]. Compared to the latter reference, the present work may be seen as a generalization to the transversely isotropic case which may represent many sedimentary rocks composed of a porous matrix with embedded solid inclusions. The fundamental assumption of the model is a scale separation between the pores (lower level) and the solid inclusion (upper level). The pores are first added in the transversely isotropic matrix (step one) and the result in a homogenized transversely isotropic porous matrix. At a second step, the mineral solid inclusions are added in the porous matrix and the result is the homogenized porous rock-like composite. In the particular case of one phase of mineral solid inclusions (the quartz) a general multi-step homogenization scheme has been presented in [31] to estimate the poroelastic properties of shale-type material and concrete type materials. Compared to the latter reference, the model presented in this paper is strongly simplified: two homogenization steps are considered instead of four homogenization steps. The two levels taken into account in the model presented in this paper correspond to the two upper levels (referred as *micro* and *macro* levels) of the general model presented in [31]. The lower level is simplified: only one class of pores is taken into account and the *argillaceous matrix* is supposed homogeneous. The chosen reference scale of the present approach is the macroscopic scale as experimental data, for porothermoelastic coefficients of rock-like composites, are actually obtained at the macroscopic scale. The uncertainty of the model is reported to the lower level. Starting from macroscopic results and basic microstructural information as pore shape, pore orientation distribution, volume fractions of constituents, the coupled porothermoelastic properties of the argillaceous matrix are obtained by back analysis. In this context, a cross property analysis (see [26,28]) may be very useful to evaluate the relevancy of a microstructural model. As an example, a similar approach has been carried out to estimate the thermal conductivity of transversely isotropic rock-like composites in [10].

The paper is organized as it follows: In Section 1, background results for Eshelby tensor approach in transversely isotropic material are recalled. In Section 2, the two-step homogenization scheme for porous rock-like composites with application to estimation of overall porothermoelastic coefficients is presented. In Section 3, numerical results are presented for a transversely isotropic porous rock.