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Extended criticality, phase spaces and enablement in biology

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ABSTRACT

This paper analyzes, in terms of critical transitions, the phase spaces of biological dynamics. The phase space is the space where the scientific description and determination of a phenomenon is given. We argue that one major aspect of biological evolution is the continual change of the pertinent phase space and the unpredictability of these changes. This analysis will be based on the theoretical symmetries in biology and on their critical instability along evolution.

Our hypothesis deeply modifies the tools and concepts used in physical theorizing, when adapted to biology. In particular, we argue that causality has to be understood differently, and we discuss two notions to do so: differential causality and enablement. In this context constraints play a key role: on one side, they restrict possibilities, on the other, they enable biological systems to integrate changing constraints in their organization, by correlated variations, in un-prestatable ways. This corresponds to the formation of new phenotypes and organisms.

1. Introduction

As extensively stressed by Weyl and van Fraassen, XXth century physics has been substituting to the concept of law that of symmetry. Thus, this concept may be "considered the principal means of access to the world we create in theories" [56].

In this text,[1] we will discuss the question of biological phase spaces in relation to critical transitions and symmetries. More precisely, we will argue, along the lines of [27,8,37], that in contrast to existing physical theories, where phase spaces are pre-given, in biology these spaces need to be analyzed as changing in unpredictable ways through evolution. This stems from the peculiar biological relevance of critical transitions and the related role of symmetry changes.

In order to understand the peculiarities of biological theorizing, we will first shortly recall the role, in physics, of "phase spaces". A phase space is the space of the pertinent observables and parameters in which the theoretical determination of the system takes place. As a result, to one point of the phase space corresponds a complete determination of the intended object and properties that are relevant for the analysis.

Aristotle and Aristotelians, Galileo and Kepler closely analyzed trajectories of physical bodies, but without a mathematical theory of a "background space". In a sense, they had the same attitude as Greek geometers: Euclid's geometry is a geometry of figures with no space. It is fair to say that modern mathematical physics (Newton) begun by the "embedding" of Kepler and Galileo's Euclidean trajectories in Descartes' spaces. More precisely, the conjunction of these spaces with Galileo's inertia gave the early relativistic spaces and their invariant properties, as a frame

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[1] This paper revises and develops early joint work with Stuart Kauffman [37].

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for all possible trajectories — from falling bodies to revolving planets.[2] In modern terms, Galileo's symmetry group describes the transformations that preserve the equational form of physical laws, as invariants, when changing the reference system.

Along these lines, one of the major challenges for a (theoretical) physicist is to invent the pertinent space or, more precisely, to construct a mathematical space which contains all the required ingredients for describing the phenomena and to understand the determination of its trajectory, if any. So, Newton's analysis of trajectories was embedded in a Cartesian space, a "condition of possibility", Kant will explain, for physics to be done. By this, Newton unified (he did not reduce) Galileo's analysis of falling bodies, including apples, to planetary orbits: Newton derived Kepler's ellipsis of a planet around the Sun from his equations. This is the astonishing birth of modern mathematical-physics as capable of predicting exactly the theoretical trajectory, once given the right space and the exact boundary conditions. But, since Poincaré, we know that if the planets around the Sun are two or more, prediction is impossible due to deterministic chaos. Even though their trajectories are fully determined by Newton–Laplace equations their non-linearity yields the absence almost everywhere of analytic solutions and forbids predictability, even along well determined trajectories at equilibrium.

As a matter of fact, Poincaré's analysis of chaotic dynamics was essentially based on his invention of the so-called Poincaré section (analyze planetary orbits only by their crossing a given plane) and by the use of momentum as a key observable. In his analysis of chaoticity, stable and unstable trajectories in the *position-momentum* phase space, nearly intersect infinitely often, in "infinitely tight meshes" and are also "folded upon themselves without ever intersecting themselves", (1892). Since then, in physics, the phase space is mostly given by all possible values of momentum and position, or energy and time. In Hamiltonian classical mechanics and in Quantum Physics, these observables and variables happen to be "conjugated", a mathematical expression of their pertinence and tight relation.[3] These mathematical spaces are the spaces in which the trajectories are determined: even in Quantum Physics, when taking Hilbert's spaces as phase spaces for the wave function, Schrödinger's equation *determines* the dynamics of a probability density and the indeterministic aspect of quantum mechanics appears when quantum measurement projects the state vector (and gives a probability, as a real number value).

It is then possible to give a broader sense to the notion of phase space. For thermodynamics, say, Boyle, Carnot and Gay-Lussac decided to focus on pressure, volume and temperature, as the relevant observables: the phase space for the thermodynamic cycle (the interesting "trajectory") was chosen in view of its pertinence, totally disregarding the fact that gases are made out of particles. Boltzmann later unified the principles of thermodynamics to a particle's viewpoint and later to Newtonian trajectories by adding the ergodic hypothesis. Statistical mechanics thus, is not a reduction of thermodynamics to Newtonian trajectories, rather an "asymptotic" unification, at the infinite time limit of the thermodynamic integral, under the novel assumption of "molecular chaos" (ergodicity). In statistical mechanics, ensembles of random objects are considered as the pertinent objects, and observables are derived as aspects of their (parameterized) statistics.

It should be clear that, while the term phase space is often restricted to a position/momentum space, we use it here in the general sense of the suitable or intended space of the mathematical and/or theoretical description of the system. In this sense the very abstract Hilbert space of complex probability densities is a phase space for the state function in Quantum Mechanics, very far form ordinary space–time.

Now, in biology, the situation is more difficult. Our claim here, along the lines of [27,10,37] is that, when considering the biologically pertinent observables, organisms and phenotypes, no conceptual nor mathematical construction of a pre-given phase space is possible for phylogenetic trajectories. This constitutes a major challenge in the study of biological phenomena. We will motivate it by different levels of analysis. Of course, our result is a "negative result", but negative results may open the way to new scientific thinking, in particular by the very tools proposed to obtain them, [43]. Our tools are based on the role of symmetries and criticality, which will suggest some possible ways out.

2. Phase spaces and symmetries

We understand the historically robust "structure of determination of physics" (which includes unpredictability) by recalling that, since Noether and Weyl, physical laws may be described in terms of theoretical symmetries in the intended equations (of the "dynamics", in a general sense, see below). These symmetries in particular express the fundamental conservation laws of the physical observables (energy, momentum, charges ...), both in classical and quantum physics. And the conservation properties allow us to compute the trajectories of physical objects as geodetics, by extremizing the pertinent functionals (Hamilton principle applied to the Langrangian functionals). It is the case even in Quantum Mechanics, as they allow to derive the trajectory of the state function in a suitable mathematical space, by Schrödinger equation.

As we said, only with the invention of an (analytic) geometry of space (Descartes), could trajectories be placed in a mathematically pre-given space, which later became the absolute space of Newtonian laws. The proposal of the more general notion of "phase space" dates of the late XIX century. Then momentum was added to spatial position as an integral component of the analysis of a trajectory, or energy to time, in order to apply the corresponding conservation properties, thus the corresponding theoretical symmetries. In general, the phase spaces are the right spaces of description in the sense that they allow one to soundly

[2] The Italian Renaissance painting invented the mathematical "background" space by the perspective, later turned into mathematics by Descartes and Desargues, see [42].

[3] One is the position and the other takes into account the mass and the change of position.

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