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# Symmetries in 4-dimensional manifolds with metric of neutral signature

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**Abstract**

This paper studies some general principles of metric (Killing) symmetry on a 4−dimensional, connected manifold $M$ admitting a metric of neutral signature. First a general survey of neutral geometry is presented and this is followed by a discussion of the generalised distribution, integral manifolds and orbits which arise from a general Lie algebra of global smooth vector fields on $M$. Next, this theory is applied to the finite-dimensional Lie algebra of global Killing vector fields on $M$ and gives a decomposition of $M$ into its various Killing orbits. The zeros of Killing vector fields and the consequent isotropies are then introduced, the latter being described in terms of subalgebras of $o(2,2)$. The concepts of *stability* and *dimensional stability* for orbits is described and theorems are proved which relate the dimension of the Killing algebra to the orbit and isotropy dimensions. Certain special subalgebras are then introduced and related to isotropies in dimensionally stable orbits. Several examples of metrics with symmetries are then presented and which reveal many of the above features. Finally, a theorem describing the consequences for the Ricci and Weyl tensors under Killing isotropy is given in terms of algebraic classifications of these tensors. A table is provided which gives full details of the above results.

## 1 Introduction

This paper is intended as a contribution to the theory of metric symmetry on a 4-dimensional, connected manifold with metric $g$ of neutral signature $(+,+,-,-)$. Section 2 will be devoted to notation and to the conventions used and to a brief survey of the neutral geometry of $(M,g)$. In section 3 the interrelated concepts of distribution, maximal integral manifold, orbit and leaf associated with a Lie algebra of global vector fields $J$ on $M$ will be summarised. In sections $4-6$ symmetry is finally introduced through a discussion of the existence of global Killing vector fields on $(M,g)$, the associated Killing algebra and orbits and