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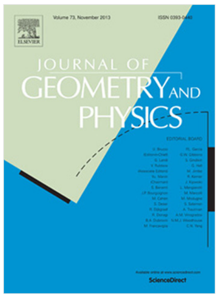

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TOWARD A GALOIS THEORY OF THE INTEGERS OVER THE SPHERE SPECTRUM

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ABSTRACT. Recent work [1, 2, 7, 18, 44] in higher algebra allows the reinterpretation of a classical description [8, 11, 31, 32] of the Eilenberg-MacLane spectrum $H\mathbb{Z}$ as a Thom spectrum, in terms of a kind of derived Galois theory. This essentially expository talk summarizes some of this work, and suggests an interpretation in terms of configuration spaces and monoidal functors on them, with some analogies to a topological field theory.

1. INTRODUCTION

1.1. Questions about an absolute base $\mathbb{F}_1 \to \mathbb{Z}$ for arithmetic (e.g. [12, 23]) evoke echoes in algebraic topology (e.g. [50, 51]), where Waldhausen's brave new rings program interprets the unit $1 \in \tilde{H}^0(S^0, \mathbb{Z})$ as a kind of ring homomorphism from the sphere spectrum S^0 to the Eilenberg-MacLane spectrum $H\mathbb{Z}$. This talk, aimed at interested non-experts, tries to summarize current thinking (and speculation) about this and related questions in homotopy theory, framed in terms of recent work of Rognes, Hess, and others on an emerging version of Galois theory in higher algebra. A now classic construction by Mahowald from the late 1970s (then at $p = 2$, though quickly generalized [31, Corollary 3.5] [11]) interprets $H\mathbb{Z}$ as a Thom spectrum (or cobordism theory); this talk describes that result in this developing language.

Some conventions: We write $\wedge$ for the (symmetric monoidal) smash product of pointed spaces, and $\wedge_R$ (e.g. $\wedge_{S^0}$) for the smash product of R-module spectra. If G is a group, the geometric realization or nerve $|[*/G]| := BG$ of the associated singleton category provides a standard model for its classifying space. $\Sigma^\infty X_+$ (or $S^0[X]$) will denote the suspension spectrum defined by an unpointed space. X_∞ will denote one-point compactification (e.g. $\mathbb{C}_\infty \cong S^2$). For a space X we use $X\langle n\rangle$ to indicate the nth level of the Whitehead tower, i.e. the space which has trivial homotopy in degrees less than *or equal to* n, and is equivalent to X elsewhere. There seems to be some disagreement in the literature about the precise meaning of this

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