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## FOUR-DIMENSIONAL PSEUDO-RIEMANNIAN G.O. SPACES AND MANIFOLDS

GIOVANNI CALVARUSO AND AMIRHESAM ZAEIM

ABSTRACT. A g.o. manifold is a homogeneous pseudo-Riemannian manifold whose geodesics are all homogeneous, that is, they are orbits of a one-parameter group of isometries. A g.o. space is a realization of a homogeneous pseudo-Riemannian manifold  $(M, g)$  as a coset space  $M = G/H$ , such that all the geodesics are homogeneous.

We prove that apart from the already classified non-reductive examples [Calvaruso, Fino and Zaeim, 2015], any four-dimensional pseudo-Riemannian g.o. manifold is naturally reductive. To obtain this result, we shall also provide a complete description up to isometries of four-dimensional pseudo-Riemannian g.o. spaces, and show explicit realizations of the four-dimensional pseudo-Riemannian naturally reductive spaces classified in [Batat, Castrillon-Lopez and Rosado, 2015].

### 1. INTRODUCTION AND BACKGROUND INFORMATION

Let  $(M, g)$  be a homogeneous pseudo-Riemannian manifold. Denoted by  $G \subset I_0(M, g)$  a connected Lie group of isometries acting transitively on  $M$ , the manifold can be identified with the pseudo-Riemannian homogeneous space  $(G/H, g)$ , where  $H$  denotes the isotropy group of the origin  $P_0 \in M$ .

A geodesic  $\gamma$  through  $P_0 \in M = G/H$  is called *homogeneous* if it is the orbit of a one-parameter subgroup. In general, the group  $G$  is not unique. Clearly, if  $\gamma$  is homogeneous with respect to some isometry group  $G$ , then it is also homogeneous with respect to the maximal connected group of isometries, but not conversely.

A homogeneous pseudo-Riemannian manifold  $(M, g)$  is said to be *reductive* if  $M = G/H$  and the Lie algebra  $\mathfrak{g}$  of  $G$  can be decomposed into a direct sum  $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$ , where  $\mathfrak{m}$  is an  $\text{Ad}(H)$ -invariant subspace of  $\mathfrak{g}$ . It is well known that when  $H$  is connected, this condition is equivalent to the algebraic condition  $[\mathfrak{h}, \mathfrak{m}] \subseteq \mathfrak{m}$ . With regard to the existence of a reductive decomposition, a fundamental difference arises between homogeneous Riemannian and pseudo-Riemannian manifolds. In fact, any homogeneous Riemannian manifold is reductive, but in dimension four and higher, there exist homogeneous pseudo-Riemannian manifolds which do not admit any reductive decomposition. Four-dimensional homogeneous non-reductive pseudo-Riemannian manifolds, both Lorentzian and of neutral signature, were classified in [23], and an explicit description of their invariant homogeneous metrics was obtained in [13].

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