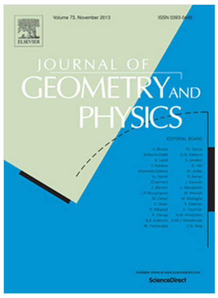
Accepted Manuscript



Heat asymptotics for nonminimal Laplace type operators and application to noncommutative tori

B. Iochum, T. Masson

PII: S0393-0440(18)30066-4
DOI: https://doi.org/10.1016/j.geomphys.2018.02.014
Reference: GEOPHY 3162

To appear in: *Journal of Geometry and Physics*

Received date : 25 September 2017
Revised date : 7 February 2018
Accepted date : 14 February 2018

Please cite this article as: B. Iochum, T. Masson, Heat asymptotics for nonminimal Laplace type operators and application to noncommutative tori, *Journal of Geometry and Physics* (2018), https://doi.org/10.1016/j.geomphys.2018.02.014

Heat asymptotics for nonminimal Laplace type operators and application to noncommutative tori

B. Iochum[a], T. Masson[a]

[a] *Centre de Physique Théorique*
Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France

Abstract

Let P be a Laplace type operator acting on a smooth hermitean vector bundle V of fiber $\mathbb{C}^N$ over a compact Riemannian manifold given locally by $P = -[g^{\mu\nu}u(x)\partial_\mu\partial_\nu + v^\nu(x)\partial_\nu + w(x)]$ where u, v^ν, w are $M_N(\mathbb{C})$-valued functions with $u(x)$ positive and invertible. For any $a \in \Gamma(\mathrm{End}(V))$, we consider the asymptotics $\mathrm{Tr}(a\, e^{-tP}) \underset{t\downarrow 0^+}{\sim} \sum_{r=0}^{\infty} a_r(a,P)\, t^{(r-d)/2}$ where the coefficients $a_r(a,P)$ can be written as an integral of the functions $a_r(a,P)(x) = \mathrm{tr}[a(x)\mathcal{R}_r(x)]$.
The computation of $\mathcal{R}_2$ is performed opening the opportunity to calculate the modular scalar curvature for noncommutative tori.

Keywords: Heat kernel, nonminimal operator, asymptotic heat trace, Laplace type operator, scalar curvature, noncommutative torus
PACS: 11.15.-q, 04.62.+v
2000 MSC: 58J35, 35J47, 81T13, 46L87

1. Introduction

As in [1], we consider a d-dimensional compact Riemannian manifold (M,g) without boundary, together with a nonminimal Laplace type operator P on a smooth hermitean vector bundle V over M of fiber $\mathbb{C}^N$ written locally as

$$P := -[\, g^{\mu\nu}u(x)\partial_\mu\partial_\nu + v^\nu(x)\partial_\nu + w(x)\,]. \tag{1.1}$$

Here $u(x) \in M_N(\mathbb{C})$ is a positive and invertible matrix valued function and v^ν, w are $M_N(\mathbb{C})$ matrices valued functions. The operator is expressed in a local trivialization of V over an open subset of M which is also a chart on M with coordinates (x^μ). This trivialization is such that the adjoint for the hermitean metric corresponds to the adjoint of matrices and the trace on endomorphisms on V becomes the usual trace tr on matrices.

For any $a \in \Gamma(\mathrm{End}(V))$, we consider the asymptotics of the heat-trace

$$\mathrm{Tr}(a\, e^{-tP}) \underset{t\downarrow 0^+}{\sim} \sum_{r=0}^{\infty} a_r(a,P)\, t^{(r-d)/2}. \tag{1.2}$$

Email addresses: bruno.iochum@cpt.univ-mrs.fr (B. Iochum), thierry.masson@cpt.univ-mrs.fr (T. Masson)