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Noncommutative Euclidean spaces

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NONCOMMUTATIVE EUCLIDEAN SPACES

MICHEL DUBOIS-VIOLETTE, GIOVANNI LANDI

ABSTRACT. We give a definition of noncommutative finite-dimensional Euclidean spaces \mathbb{R}^n . We then remind our definition of noncommutative products of Euclidean spaces \mathbb{R}^{N_1} and \mathbb{R}^{N_2} which produces noncommutative Euclidean spaces $\mathbb{R}^{N_1+N_2}$. We solve completely the conditions defining the noncommutative products of the Euclidean spaces \mathbb{R}^{N_1} and \mathbb{R}^{N_2} and prove that the corresponding noncommutative unit spheres $S^{N_1+N_2-1}$ are noncommutative spherical manifolds. We then apply these concepts to define “noncommutative” quaternionic planes and noncommutative quaternionic tori on which acts the classical quaternionic torus $T_{\mathbb{H}}^2 = U_1(\mathbb{H}) \times U_1(\mathbb{H})$.

Dedicated to Alain Connes for his 70th birthday
(i.e. dix fois “l’âge de raison” !)

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