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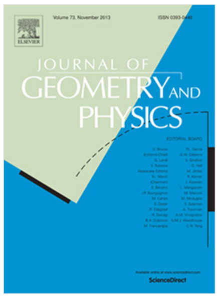

Hamiltonian stability for weighted measure and generalized Lagrangian mean curvature flow

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Hamiltonian stability for weighted measure and generalized Lagrangian mean curvature flow

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Abstract

In this paper, we generalize several results for the Hamiltonian stability and the mean curvature flow of Lagrangian submanifolds in a Kähler-Einstein manifold to more general Kähler manifolds including a Fano manifold equipped with a Kähler form $\omega \in 2\pi c_1(M)$ by using the method proposed by T. Behrndt [5]. Namely, we first consider a weighted measure on a Lagrangian submanifold L in a Kähler manifold M and investigate the variational problem of L for the weighted volume functional. We call a stationary point of the weighted volume functional f-minimal, and define the notion of Hamiltonian f-stability as a local minimizer under Hamiltonian deformations. We show such examples naturally appear in a toric Fano manifold. Moreover, we consider the generalized Lagrangian mean curvature flow in a Fano manifold which is introduced by Behrndt and Smoczyk-Wang. We generalize the result of H. Li, and show that if the initial Lagrangian submanifold is a small Hamiltonian deformation of an f-minimal and Hamiltonian f-stable Lagrangian submanifold, then the generalized MCF converges exponentially fast to an f-minimal Lagrangian submanifold.

Keywords: Lagrangian submanifolds, toric Fano manifolds, Hamiltonian stability, Generalized Lagrangian mean curvature flow
2010 MSC: Primary 53D12, Secondary 53C44.

1. Introduction

Let (M, ω) be a symplectic manifold of $\dim_{\mathbb{R}} M = 2n$, where ω is the symplectic form. An n-dimensional submanifold L in M is called *Lagrangian* if $\omega|_L = 0$, and Lagrangian submanifolds are investigated by several geometric motivations. When M admits a Riemannian metric, variational problems for Lagrangian submanifolds are of particular interests. For example, Harvey-Lawson introduced the notion of special Lagrangian submanifolds in Calabi-Yau manifolds [16], which is a submanifold with volume-minimizing property with respect to the Ricci-flat Kähler metric. In the Calabi-Yau setting, the special condition of L is actually equivalent to the *minimality* of L, namely, L is a critical point of the volume functional, and the minimality is described by vanishing mean curvature. See [17] for further developments and some construction methods of special Lagrangian submanifold in a Calabi-Yau manifold.

On the other hand, the situation is different when M admits a Kähler structure with Kähler metric g of positive Ricci curvature. In fact, Lawson-Simons [24] proved that, in the complex projective space $\mathbb{C}P^n$ with the Fubini-Study metric, a stable submanifold for the volume functional is actually a complex submanifold, and hence, any minimal Lagrangian submanifold in $\mathbb{C}P^n$ is never stable in the usual sense because the Lagrangian condition implies the tangent space of L does not contain any complex distribution. In such a remarkable situation, Y.-G. Oh studied a variational problem for Lagrangian submanifold in

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