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SYMMETRIES AND CONSERVATION LAWS OF A NONLINEAR SIGMA MODEL WITH GRAVITINO

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Abstract. We study the symmetries and invariances of a version of the action functional of the nonlinear sigma model with gravitino, as considered in [12]. The action is invariant under rescaled conformal transformations, super Weyl transformations and diffeomorphisms. In particular cases the functional possesses a degenerate supersymmetry. The corresponding conservation laws lead to a geometric interpretation of the energy-momentum tensor and supercurrent as holomorphic sections of appropriate bundles.

1. Introduction

The main motivation for the introduction of the two-dimensional supersymmetric nonlinear sigma model in quantum field theory, or more specifically supergravity and superstring theory, are its symmetries, see for instance [2, 5, 6, 10]. Furthermore, as argued in [14], the functional is determined by its symmetries together with suitable bounds on the order of its Euler–Lagrange equations. While supersymmetric models are usually formulated using anticommuting variables, in [12] an analogue of the two-dimensional nonlinear supersymmetric sigma model using only commuting variables was introduced. Here we would like to give a detailed geometric account of the symmetries of this purely commutative model.

We briefly recall the two-dimensional nonlinear sigma model constructed in [12]. Let M be a Riemann surface and let (N, h) be a Riemannian manifold. In the classical nonlinear sigma model, the action functional is given by the Dirichlet energy functional which is defined for a map $\phi\colon M \to N$ and a Riemannian metric g on M. In our model we need to take also their superpartners into consideration. These superpartners are geometrically formulated via suitable spinor fields. To be more precise, given the Riemannian metric g, we fix a spin structure $\xi\colon P_{\mathrm{Spin}}(M, g) \to P_{\mathrm{SO}}(M, g)$. An irreducible representation of the Clifford algebra $\mathrm{Cl}_{0,2}$ induces the real spin representation $\mu\colon \mathrm{Spin}(2) \to \mathrm{GL}(V)$, where V is a representation space of real dimension four. The associated spinor bundle $S_g \coloneqq P_{\mathrm{Spin}}(M, g) \times_\mu V$ is equipped with the canonical spinor metric g_s and the spin connection ∇^s induced by the Levi-Civita connection on TM. Choosing an isomorphism between the vector spaces V and $\mathrm{Cl}_{0,2}$ we get a Clifford map $\gamma\colon TM \to \mathrm{End}(S_g)$ which satisfies the Clifford relation

$$\gamma(X)\gamma(Y) + \gamma(Y)\gamma(X) = -2g(X, Y), \qquad \forall X, Y \in \Gamma(TM).$$

Sections of S_g will be referred to as (pure) spinors, which describe matter fields with half-integer spins in physics. The spin Dirac operator $\partial\!\!\!/_g \coloneqq \gamma(e_\alpha)\nabla^s_{e_\alpha}\colon \Gamma(S_g) \to \Gamma(S_g)$ is a first-order elliptic differential operator, which is self-adjoint if M is closed.

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Key words and phrases. nonlinear sigma model, gravitino, Noether's theorem, symmetry, energy-momentum tensor, supercurrent, Dirac-harmonic map.

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