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Interaction between non-parallel dislocations in piezoelectrics

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A B S T R A C T

The total interaction force $\mathbf{F}^{12}$ between two crossing (non-intersecting) straight dislocations is found and analyzed for the three types of piezoelectric media of unrestricted anisotropy: an unbounded body, an infinite plate and a half-infinite body. In the latter two cases the dislocations are supposed to be parallel with the surfaces, which are in turn implied to be mechanically free of tractions and electrically closed (metalized). The found force $\mathbf{F}^{12}$ is orthogonal to the parallel planes, P and Q, containing the crossing dislocations. In an unbounded medium the value $\mathbf{F}^{12}$ proves to be independent of the distance between P and Q. On the other hand, it depends on directions of the dislocations and on their Burgers vectors: the force $\mathbf{F}^{12}$ may be either attractive or repulsive. In a plate the interaction becomes sensitive to dislocation positions $y^{(1,2)}$ with respect to the surfaces. Only in the situations, when dislocations are much closer to each other than to the both surfaces, their interaction may be approximately described by the solution for an unbounded medium. Otherwise, corrections arising from the image forces due to the plate surfaces become essential. The dislocation in the vicinity of a surface strongly acts on its counterpart only until the latter situates even closer to the same surface than the first one. When the second dislocation leaves this narrow zone, the interaction force on it abruptly decreases to a very small level. With an increase in the thickness of the plate, this behavior becomes more and more pronounced. In a half-infinite medium the interaction between the dislocations is exactly described by a Heaviside step-like dependence $F^{12} \propto H(y^{(1)} - y^{(2)})$ valid for any $y^{(1,2)}$. It is shown that we deal here with an analog of the plane capacitor effect.

1. Introduction

Dislocations in piezoelectrics produce in the vicinity of their cores not only high mechanical stresses but also singular electric fields, which might course inadmissible disturbances in functioning of modern supersensitive electronic devices. Interactions between dislocations in piezoelectric crystals and long-range electroelastic fields created by dislocations may be of crucial importance for solid elements working under conditions (e.g. at high temperatures), which might provoke a loss of stability of the real structure of a material due to a sudden relaxation of internal stresses. Such processes are determined by elementary displacements of individual dislocations through random distributions of other crystal defects. Under not very high temperatures, the main role in dislocation pinning is played by point defects (interstitial and impurity atoms, vacancies, etc.). Intersections of a moving dislocation with other dislocations are much more seldom events (a typical distance between point defects in a slip plane is ∼0.1 μm, whereas an average distance between dislocations is normally 10–100 μm. And the both types of defects create energy barriers for a dislocation motion of comparable height.

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On the other hand, as was proved by Kroupa [1] (for an infinite isotropic medium) and by Orlov and Indenbom [2,3] (for an arbitrary anisotropic medium), the total interaction force between non-parallel dislocations is independent of the distance between them. Thus, in contrast to point defects, which interact with a dislocation in a contact manner, most of dislocations in a crystal simultaneously contribute to the combined force on anyone of them. In the end of this paper we shall demonstrate that this force may be very substantial and at high enough temperatures might cause a motion of even aged dislocations.

In strong piezoelectrics an electric contribution to elastic stresses may be comparable with a customary Hooke's strain influence. Thus, the force of interaction between dislocations found in [1–3] should be further extended for a piezoelectric medium. The first studies of dislocation fields in piezoelectrics were accomplished for straight dislocations in unbounded media [4–6]. Then the results of these works were generalized to piezoelectric plates [7,8] and layer-substrate structures [9]. Considerations in [7–9] of 2D fields of straight dislocations were based on the 8D formalism [10] extending the sextic Stroh theory [11] to a description of media with piezoelectric coupling. In [12] a 4D variant of this formalism was used for a reformulation of a number of classical results of dislocation theory to a description of electroelastic fields of arbitrary curved dislocations in unbounded piezoelectric media of unrestricted anisotropy.

In this paper we shall extend for piezoelectrics the other classical result of dislocation theory: the mentioned above theorem of Orlov and Indenbom [2,3] about the integral interaction force between two non-parallel straight dislocations in an infinite anisotropic elastic medium. We shall consider a series of piezoelectric structures: infinite and semi-infinite media and an infinite plate. In all cases the crossing (non-intersecting) dislocations will be supposed to be parallel to surfaces.

Though the fields of both straight dislocations are two-dimensional, for non-parallel dislocations the description of their interaction generally requires solving a 3D problem. However, as was shown by Orlov and Indenbom [2], the finding of the *integral* interaction of crossing dislocations may be reduced to a simple 1D problem saving one from cumbersome direct calculations. Fortunately, the same arguments are equally applicable to all our piezoelectric structures, as well as to their purely elastic analogues not analyzed in [2,3].

2. Statement of the problem

2.1. Generalized 4D dislocations in piezoelectrics and their modified description

Consider a piezoelectric medium with the elastic moduli c_{ijkl}, the piezoelectric moduli e_{ikl} and the permittivity tensor ε_{ik}. In such medium the stress tensor σ_{ij} and the electric displacement D_i are related to the elastic distortion u_{kl} and the electric field E_k by the constitutive equations

$$\sigma_{ij} = c_{ijkl}u_{kl} - e_{kij}E_k, \quad D_i = e_{ikl}u_{kl} + \varepsilon_{ik}E_k, \tag{1}$$

and the corresponding equilibrium equations are given by

$$\sigma_{ij,i} = 0, \quad D_{i,i} = 0. \tag{2}$$

In Eqs. (1) and (2) and in the forthcoming, as usual, repeated indices imply summation and the notation $\ldots_{,l} \equiv \partial/\partial x_l$ is accepted.

In the presence of a dislocation one should distinguish between elastic (u_{kl}), plastic (intrinsic) (u^0_{kl}) and total ($u^{\mathrm{T}}_{kl} = u_{l,k}$) distortions which are related to each other through the Kroener equation [13]

$$u^{\mathrm{T}}_{kl} = u_{kl} + u^0_{kl}. \tag{3}$$

By definition,

$$u^0_{kl}(\mathbf{r}) = -n^S_k b_l \delta\left[\left(\mathbf{r} - \mathbf{r}^S\right)\cdot \mathbf{n}^S\right], \tag{4}$$

where b_l are components of the Burgers vector $\mathbf{b}$ of the dislocation, the vector $\mathbf{n}^S$ is the unit normal at the point $\mathbf{r}^S$ to the arbitrarily chosen surface S bounded by the dislocation line and $\delta(y)$ is the Dirac delta function.

In piezoelectrics one can generalize a traditional dislocation defined by a jump of the displacement vector $\mathbf{u}(\mathbf{r})$ $\left(\mathbf{u}^{S^-} - \mathbf{u}^{S^+} = \mathbf{b}\right)$ on the arbitrary surface S bounded by the dislocation line. According to [12], the jump of potential $\Delta\varphi$ at the same cut surface S leads to the source electric field quite similar to (4),

$$\mathbf{E}^0(\mathbf{r}) = -\Delta\varphi\mathbf{n}^S\delta\left[\left(\mathbf{r} - \mathbf{r}^S\right)\cdot \mathbf{n}^S\right], \tag{5}$$

which in analogy with (3) determines the total electric field

$$\mathbf{E} = \mathbf{E}^0 + \mathbf{E}' = \mathbf{E}^0 - \nabla\varphi. \tag{6}$$

Thus, one can introduce the electro-elastic line defect – a 4D dislocation – characterized by a double jump at the surface S of both the displacement, $\Delta\mathbf{u} = \mathbf{b}$, and the electric potential, $\Delta\varphi$, i.e. by the "Burgers" 4-vector

$$\mathbf{B} = \begin{pmatrix} \mathbf{b} \\ \Delta\varphi \end{pmatrix}. \tag{7}$$