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# Micromechanical analysis of magneto-electro-thermo-elastic composite materials with applications to multilayered structures

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ABSTRACT

The method of asymptotic homogenization was used to analyze a periodic magnetoelectric smart composite structure consisting of piezoelectric and piezomagnetic phases. The asymptotic homogenization model is derived, the governing equations are determined and subsequently general expressions called unit-cell problems that can be used to determine the effective elastic, piezoelectric, piezomagnetic, thermal expansion, dielectric, magnetic permeability, magnetoelectric, pyroelectric and pyromagnetic coefficients are presented. The latter three sets of coefficients are particularly interesting in the sense that they represent product or cross-properties; they are generated in the macroscopic composite via the interaction of the different phases, but may be absent from the constituents themselves. The derived expressions pertaining to the unit-cell problems and the resultant effective coefficients are very general and are valid for any 3-D geometry of the unit cell. The model is illustrated by means of longitudinally-layered smart composites consisting of piezoelectric (Barium Titanate) and piezomagnetic (Cobalt Ferrite) constituents. Closed-form expressions for the effective properties are derived and the results are plotted vs. the volume fraction of the piezoelectric phase. Pertaining to the product properties of this particular magnetoelectric laminate, it is observed that the effective pyroelectric and pyromagnetic coefficients attain a maximum value at a $BaTiO_3$ volume fraction of 0.5 and maximum values for the magnetoelectric coefficients at a $BaTiO_3$ volume fraction of 0.4. Likewise, the maximum value of a magnetoelectric figure of merit (characterizing efficiency of energy conversion in longitudinal direction) is also attained at a volume fraction of 0.4.



## 1. Introduction

Significant advancements in the production of composites coupled with emerging technologies in the fields of sensors and actuators have permitted the integration of smart composites into such fields as the aerospace industry, civil engineering, transportation and marine engineering to compliment and in some case replace more traditional materials. A unique class of smart composites consisting of piezoelectric and piezomagnetic phases has attracted particular interest in recent years on account of both their significant potential and their interesting properties. In particular, the coupling between the material properties of the different constituents can give rise to new unique effects that characterize the macroscopic composite but are absent from the constituents themselves. Examples of such properties known as cross or product properties are the magnetoelectric, pyroelectric and pyromagnetic properties.

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In a composite of piezoelectric and piezomagnetic or magnetostrictive phases for example, applying a magnetic field induces a mechanical deformation of the piezomagnetic constituent. In turn, this strain is transferred to the piezoelectric phase which induces an electric polarization. Ryu et al. [1], prepared magnetoelectric laminates by stacking and bonding PZT and Terfenol-D disks. By means of different processing and geometric parameters, they obtained a magnetoelectric voltage coefficient more than 30 times larger than the previously reported highest values. The reverse effect exists as well; that is, applying an electric field induces strain in the piezoelectric material which is in turn transferred to the magnetostrictive phase resulting in magnetization.

Pyroelectricity is another interesting example of a cross property, see Newnham et al. [2]. Application of heat to a composite results in thermal expansion and in turn to electric polarization when the mechanical strain is transferred to the piezoelectric phase. Even if the individual constituents of the composite do exhibit intrinsic pyroelectricity, the secondary product effect produced due to the coupling of the different phases can make a significant contribution, see Newnham et al. [2]. Nan et al. [3], represent product properties in composites in the following manner:

$$\text{Magnetoelectric effect} = \frac{\text{Magnetic}}{\text{Mechanical}} \times \frac{\text{Mechanical}}{\text{Electric}}$$
$$\text{Pyroelectric effect} = \frac{\text{Thermal}}{\text{Mechanical}} \times \frac{\text{Mechanical}}{\text{Electric}}$$
$$\text{Pyromagnetic effect} = \frac{\text{Thermal}}{\text{Mechanical}} \times \frac{\text{Mechanical}}{\text{Magnetic}}$$

Interest in product properties in general and magnetoelectricity in particular has sparked a broad interest in the fabrication, processing, characterization and modeling of pertinent composites. With regards to micromechanical modeling, Bichurin et al. [4] investigated the magnetoelectric effect in ferromagnetic/piezoelectric multilayer composites by employing an averaging procedure to derive effective material properties. In what amounts to a two-step process, the constitutive properties of the individual phases are first written down (assuming linear or linearized behaviour) and then the composite is treated as a macroscopically homogeneous entity with a new set of constitutive equations characterized by an effective set of coefficients that now include the coupled magnetoelectric effect. The solution of the constitutive equations from the two steps gives the effective coefficients. An extension of this work, see Bichurin et al. [5], was carried out for magnetostrictive-piezoelectric nanostructures. The authors first considered only extensional deformation of the composite and subsequently also accommodated flexural deformation the latter naturally giving rise to position-dependent magnetoelectric constants, see also Harshe et al. [6]. It should be mentioned at this point that in the case of nanocomposites, particularly those of 0–3 connectivity involving particles embedded in a matrix, an interphase layer surrounds the nanoparticles essentially rendering the bond between them and the matrix non-perfect. Because these interphase layers are of comparable size to the nanoparticles themselves they will naturally affect (especially for higher particle volume fractions) all macroscopic properties of the composite including the magnetoelectric coefficients. One method for handling this issue, see Sevostianov and Kachanov [7], is to homogenize the interphase layer, obtain a homogeneous inclusion with an intermediate radius and then analyse the resultant composite using, for example, the effective field method of Kanaun–Levin [8].

Harshe et al. [6] and Avellaneda and Harshe [9] obtained the effective magnetoelectric coefficient and figures of merit for 2–2 piezoelectric/magnetostrictive multilayer composites for mechanically free and clamped composites by assuming zero electrical resistance of the magnetostrictive layers (reasonable from a practical point of view), and electroded top and bottom surfaces of the outer piezoelectric layers (which would ensure poling of these layers and thus maximize the piezoelectric effect). Srinivasan et al. [10] applied this model to Nickel–Ferrite–PZT composites and compared the results with experimental data. The two approaches showed an excellent degree of conformance. Benveniste [11] examined piezoelectric/piezomagnetic composites with long fibers embedded in a continuous matrix with both constituents transversely isotropic. Huang and Kuo [12] developed a general model pertaining to magnetoelectric composites consisting of piezomagnetic inclusions in a piezoelectric matrix. The inclusions were modeled as ellipsoidal which renders the model applicable to a wide range of inclusion geometries including spheres, thin flakes and continuous fibers. The material tensors obtained are similar to the familiar Eshelby elastic tensors, see Eshelby [13]. Nan et al. [14] used the Green's Function technique to examine the magnetoelectric effect in composites with ferromagnetic rare-earth-iron alloys embedded in ferroelectric polymers. Other work can be found in Li and Dunn [15], Aboudi [16], Lee et al. [17], and others.

Liu et al. [18] employed the finite-element technique to examine piezoelectric/magnetostrictive composites with the lateral surfaces of rectangular slabs of the two components bonded together. Their work predicts a significant enhancement of the magnetoelectric output via the optimization of the composite's dimensions and in particular the thickness. A similar methodology was later applied to piezoelectric/magnetostrictive two-layer laminate, see Liu et al. [19]. As with the previous work, it is shown that the magnetoelectric output can be significantly enhanced by optimizing the thickness of the two layers. Lee et al. [20] developed a finite element-based model of periodic 2-phase composites consisting of piezoelectric fibers embedded in a piezomagnetic matrix, and 3-phase composites consisting of piezoelectric and piezomagnetic fibers embedded in an elastic matrix. The model is essentially based on micromechanically averaging the periodicity cell (unit cell) of the structures and permits the determination of their effective dielectric, magnetic, mechanical as well as the coupled-field properties. Of particular interest in the modeling of two-phase and three-phase composites is the work of Tang and Yu [21,22] who employed the variational asymptotic method to develop a micromechanical model pertaining to periodic smart