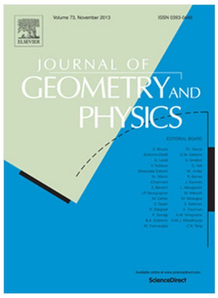
Accepted Manuscript



Rigidity of complete generic shrinking Ricci solitons

Yawei Chu, Jundong Zhou, Xue Wang

PII: S0393-0440(17)30288-7
DOI: https://doi.org/10.1016/j.geomphys.2017.11.008
Reference: GEOPHY 3107

To appear in: *Journal of Geometry and Physics*

Received date : 7 December 2016
Revised date : 7 June 2017
Accepted date : 13 November 2017

Please cite this article as: Y. Chu, J. Zhou, X. Wang, Rigidity of complete generic shrinking Ricci solitons, *Journal of Geometry and Physics* (2017), https://doi.org/10.1016/j.geomphys.2017.11.008

Rigidity of complete generic shrinking Ricci solitons

Yawei Chu, Jundong Zhou, Xue Wang

Abstract

Let (M^n, g, X) be a complete generic shrinking Ricci soliton of dimension $n \geq 3$. In this paper, by employing curvature inequalities, the formula of X-Laplacian for the norm square of the trace-free curvature tensor, the weak maximum principle and the estimate of the scalar curvature of (M^n, g), we prove some rigidity results for (M^n, g, X). In particular, it is showed that (M^n, g, X) is isometric to $\mathbb{R}^n$ or a finite quotient of $\mathbb{S}^n$ under a pointwise pinching condition. Moreover, we establish several optimal inequalities and classify those shrinking solitons for equalities.

2000 Mathematics Subject Classification: 53C24, 53C20.

Key words and phrases: Rigidity, Generic Ricci soliton, Weak maximum principle, X-Laplacian, Pinching.

1 Introduction

Let (M^n, g) $(n \geq 3)$ be an n-dimensional Riemannian manifold with metric g and X a smooth vector field on M^n. (M^n, g, X) is said to be a *generic Ricci soliton* if there exists a constant $\lambda \in \mathbb{R}$ such that

$$\mathrm{Ric} + \frac{1}{2}\mathscr{L}_X g = \lambda g, \tag{1.1}$$

where Ric and $\mathscr{L}_X g$ denote respectively the Ricci tensor and the Lie derivative of g in the direction of X, and the constant λ is sometimes called the *soliton constant*. The soliton is shrinking, steady or expanding if $\lambda > 0$, $\lambda = 0$ or $\lambda < 0$, respectively. When X is the gradient of a smooth function f on M^n, the soliton is called a *gradient Ricci soliton* and (1.1) becomes

$$\mathrm{Ric} + \mathrm{Hess}\, f = \lambda g. \tag{1.2}$$

Note that when X or ∇f is a Killing vector field, equations (1.1) and (1.2) reduce to the Einstein equation. In particular, when $X = 0$, or f is a constant, the soliton is called *trivial*.

Since the seminal works of Hamilton [14] and Perelman [23], there has been increasing interest in the study of Ricci solitons, which are correspond to self-similar solutions of Hamilton's Ricci flow [14]. Actually, Ricci solitons are also of interest to physicists and are sometimes called *quasi-Einstein metrics* in physics literature (see e.g., [12]), and can be viewed as fixed points of a dynamical system on the space of Riemannian metrics modulo diffeomorphisms and scalings.

In nearly three decades, by taking advantage of the weighted manifold structure $(M, g, \mathrm{e}^{-f}\mathrm{d\,vol})$, considerable progress in the research of gradient shrinking Ricci solitons has been made, see e.g., [2], [3], [4], [5], [7], [10], [13], [16], [17], [21], [22], [24], [25], [27], [28], among many others. On the other hand, when X is not necessarily a gradient form of a function, it is known that generic