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CV BEM for 2D transient thermo-(poro-) elastic problems concerning with blocky systems with singular points and lines of discontinuities

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ABSTRACT

This paper presents a new method for solving transient 2D thermo-(poro-) elastic problems involving blocky systems with singular points and lines of discontinuity. The efficiency and accuracy of the method arise from using complex variables (CV) and solving a problem in two separate stages. In the first stage, CV-BEM is combined with the dual reciprocity method (DRM) for transient heat (liquid) flow to obtain, as a solution, the sum of (i) a "quasi-steady" part (which may account for discontinuities and singular points) and (ii) a smooth unsteady part which is a linear combination of smooth radial basis functions (RBF). These parts, found by integrating a system of ordinary differential equations, are stored in array, which contains values for each small integration step. From these arrays, data for selected time instances, which are of interest for thermo-(poro-) elastic analysis, are used in the second stage to find stresses and stress intensity factors (SIFs) at the instances of interest. The second stage solution employs CVH-BEM for blocky systems with discontinuities and singular points: a common code of the CVH-BEM is complemented with evaluation of two pairs of addends on the right hand side of the boundary integral equation solved; one of the addends is the sum of well-known terms for the "quasi-steady" part, while the other is a linear combination of particular solutions corresponding to a standard RBF. The particular solution needed is found for the Gaussian RBF in a simple analytical form by using the CV. The efficiency and accuracy of the method are illustrated by studying stresses and stress intensity factors in a square plate with a crack under thermal shock applied either to the plate sides, or to the crack surfaces. An interesting and not obvious effect is revealed: it appears that under thermal shock on the crack surfaces, the flux intensity factor is actually independent of the crack length for short time instances, which results in moderate stress intensity factors after the thermal shock.

1. Introduction

Transient thermo-(poro-) elastic problems are of interest for many applications (see, e.g. Refs. [1,2]). However, analytical results exist for only a few configurations (see, e.g. Ref. [3]). Thus, numerical techniques such as the finite element method (FEM) and the boundary element method (BEM) have been used to obtain solutions. These problems become quite involved when it is necessary to account for stress singularities at crack tips, corner points and common apexes of grains (e.g. Refs. [4–10]). In these cases, BEM offers advantages, such as easily accounting for the asymptotic behaviour of fields at singular

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points, and simple modeling of crack growth. This method has been employed in various formulations including a volume based approach [6], time-domain boundary only formulation [8], sub-region technique for symmetric crack problems [9], dual BEM for arbitrary cracks [10]. So far, these BEM approaches have been used in real variables applicable to both 2D and 3D problems. Meanwhile, in 2D, the complex variable (CV) formulations provide further advantages, especially when solving problems for blocky systems with cracks, pores, inclusions and multi-wedge points (see, e.g. Refs. [11–13]). Therefore, it is reasonable to extend these advantages to 2D transient problems.

For uncoupled transient problems, the extension is facilitated by the fact that a transient flow problem may be solved independently of a thermo-(poro-) elastic problem, and the latter includes the transient terms only as specific time-dependent body forces. Consequently, if these forces are found in advance, the common CV-BEM may be employed by using the superposition of a particular solution accounting for the known "body-forces" and complimentary solution satisfying homogeneous equations of the static elasticity. A CV form of such a solution is given in Ref. [14]. Its advantage consists in using contour-only integrals from the time-dependent potential and the flux. Still, the suggested solution, convenient for smooth fields in a simply-connected region, becomes rather complicated when applied to discontinuous and singular fields or/and to a multi-connected region. As described in the following sections, we overcome the shortcoming by representing the transient potential (temperature or pressure) as the sum of a "quasi-steady" part, which accounts for the stated complicating factors, and a smooth non-steady part. For the "quasi-steady" part, the extension of the common CV-BIE [11,12] to the steady thermo-(poro-) elastic problems derived in [15] is available. For the smooth non-steady part, the particular solution of Ref. [14] may be easily built, especially when this part is given analytically as a linear combination of simple standard functions. Such representation is available when the dual reciprocity method (DRM) is used for solving a transient flow problem.

The required decomposition of the transient flow fields is suggested and employed in a recent paper by the authors of this paper [16]. Then, as shown below, by using the Gauss-type radial basis functions (RBF), we obtain a simple particular solution for displacements and stresses as standard holomorphic functions of the complex coordinate. A particular solution, corresponding to a linear combination of the RBF, is a similar linear combination of the standard solutions. Once obtained, it is a routine technique to include the solution in the common CV-BEM. This drastically simplifies finding not only the boundary values of tractions and displacements and SIFs at singular points but stresses at arbitrary points within the region, as well.

On the whole, the method of the present paper is as follows. A thermo-(poro-) elastic problem is solved in two separate stages. The first stage consists of finding the two parts (quasi-steady and smooth unsteady) of a transient heat or fluid transfer problem by using the CV dual reciprocity BEM with the Gauss-type RBF. In this stage, for each small-time step of the integration over time, we obtain (i) the potential and its normal derivative at the contour, and (ii) the coefficients of a linear combination of the RBF representing the pseudotemperature. Those data, which correspond to time instances of interest, are used in the second stage. For each of the selected time instances, the second stage solution employs the complex variable hypersingular (CVH) BEM, complemented with evaluation of two pairs of addends on the right hand side. One of them is the sum of well-known terms for the "quasi-steady" part. The other is a linear combination of standard particular solutions corresponding to the chosen RBF, with the coefficients found in the first stage. As a result, for each time instance, we find displacements and tractions on the boundary, SIFs at singular points and, if needed, stresses, strains and displacements at internal points.

The methods used in the two stages define the efficiency and accuracy of the overall method. Under properly chosen parameters of the DRM, quite accurate and stable results may be efficiently obtained. The efficiency is additionally facilitated by the complete separation of the stages, which may be performed at different computers at different times. The results of the first stage may be repeatedly used to find stresses at new time instances and at new points of a region. Actually, the separation makes the best of uncoupled nature of the problem.

Obviously, separation of the two stages and employing DRM in the first stage to distinguish the quasi-steady and smooth unsteady parts are also of use when solving 2D and 3D problems in real variables. The problem of finding a standard particular solution, generated by a standard RBF, is easily solved in 3D for the RBF of the Gauss-type. We shall not dwell on this option, but leave it for further work. Rather, we illustrate the efficiency and accuracy of the suggested method by considering the 2D problems of thermal shock on (i) the sides of a square plate with a straight crack, and (ii) the surfaces of the crack. The first problem allows us to compare the results with those published in Refs. [7,9,10]. The solution of the second problem reveals an interesting and not obvious feature: the thermal shock applied to the crack surfaces does not lead to drastic growth of the SIF at time instances very close to the moment of the shock.

2. Problem formulation

Consider a blocky system with pores, inclusions and lines of discontinuity such as cracks, thin inclusions and contacts of blocks (Fig. 1). The blocks and inclusions are assumed to be homogenous, linear elastic and isotropic. Denote N the total number of homogeneous sub-regions (blocks and inclusions). In the Cartesian coordinates x_k (k = 1, 2, 3 in 3D; k = 1, 2 in 2D), the equations of uncoupled thermo-(poro-) elasticity are the diffusion equation

$$\frac{\partial^2 T}{\partial x_k^2} - \frac{1}{\beta}\frac{\partial T}{\partial t} = 0 \quad j = 1, \ldots, N \tag{1}$$

and the elasticity equations