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HYPERBOLIC SHIRTS FIT A 4-BODY PROBLEM

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Abstract. Consider the equal mass planar 4-body problem with a potential corresponding to an inverse *cube* force. The Jacobi-Maupertuis principle reparametrizes the dynamics as geodesics of a certain metric. We compute the curvature of this metric in the reduced space on the collinear and parallelogram invariant surfaces and observe some dynamical consequences.

1. Introduction

The classical N-body problem concerns the motions of N point masses subject to an attractive force proportional to the inverse square of the relative distances between the masses. Poincaré noticed in [10] (see as well [6]) that variational methods work better for 'strong forces' meaning proportional to the inverse cube or a larger power of the relative distances, due to the fact that the action becomes infinite as one approaches collisions. Consequently, for strong force N-body problems it is easy to show the existence of periodic orbits by minimizing the action over almost any free homotopy class. Montgomery [8] studied the inverse cube planar problem with three equal masses using the Jacobi-Maupertuis variational principle (see eq. 4 below) and obtained uniqueness results leading to some symbolic dynamics. Recently research on this subject has been a topic of increasing interest, see for instance [5, 11, 7, 8]. Here we examine this approach for the restricted problems of 4-bodies in a collinear configuration or 4-bodies in a parallelogram configuration. Chen in [4] observed that the shape space for 4-bodies in a parallelogram configuration (figure 1) is, like for the planar 3-body problem, a punctured sphere (see figures 2, 4) and used this to great advantage in finding new choreography type solutions to the Newtonian 4-body problem. Our main result on Chen's parallelogram shape space with an inverse cube potential is that almost every free homotopy class is realized uniquely upto symmetry by a periodic solution.

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