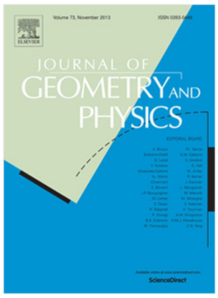
Accepted Manuscript



More exact solutions of the constant astigmatism equation

Adam Hlaváč

PII: S0393-0440(17)30212-7
DOI: http://dx.doi.org/10.1016/j.geomphys.2017.09.003
Reference: GEOPHY 3067

To appear in: *Journal of Geometry and Physics*

Received date : 4 July 2017
Revised date : 11 September 2017
Accepted date : 13 September 2017

More exact solutions of the constant astigmatism equation

Adam Hlaváč

Mathematical Institute in Opava, Silesian University in Opava, Na Rybníčku 1, 746 01 Opava, Czech Republic. *E-mail*: Adam.Hlavac@math.slu.cz

Abstract. By using Bäcklund transformation for the sine-Gordon equation, new periodic exact solutions of the constant astigmatism equation $z_{yy} + (1/z)_{xx} + 2 = 0$ are generated from a seed which corresponds to Lipschitz surfaces of constant astigmatism.

1. Introduction

In this paper, we construct new exact solutions of the *constant astigmatism equation (CAE)*

$$z_{yy} + \left(\frac{1}{z}\right)_{xx} + 2 = 0, \qquad (1)$$

the Gauss equation for *constant astigmatism surfaces* immersed in Euclidean space. These surfaces are defined by the condition $\rho_2 - \rho_1 = \mathrm{const} \neq 0$, where ρ_1, ρ_2 are the principal radii of curvature.

It is well known [5, 6, 15, 16, 17, 22] that evolutes (focal surfaces) of constant astigmatism surfaces are pseudospherical, i.e. with constant negative Gaussian curvature. Conversely, involutes of pseudospherical surfaces corresponding to parabolic geodesic net are of constant astigmatism.

After a century of oblivion, constant astigmatism surfaces reemerged in 2009 in the work [2] concerning the systematic search for integrable classes of Weingarten surfaces. In the paper, the surfaces were given a name and Equation (1) was derived for the first time. The geometric connection between surfaces of constant astigmatism and pseudospherical surfaces provides a transformation [2, Sect. 6 and 7] between the CAE and the famous *sine-Gordon equation*

$$\omega_{\xi\eta} = \sin\omega, \qquad (2)$$

which is the Gauss equation of pseudospherical surfaces parameterized by Chebyshev–asymptotic coordinates.

To our best knowledge, the list of known exact solutions of the CAE is exhausted by: