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Jong Taek Cho, Takahiro Hashinaga, Akira Kubo, Yuichiro Taketomi, Hiroshi Tamaru

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REALIZATIONS OF SOME CONTACT METRIC MANIFOLDS AS RICCI SOLITON REAL HYPERSURFACES

JONG TAEK CHO, TAKAHIRO HASHINAGA, AKIRA KUBO, YUICHIRO TAKETOMI, AND HIROSHI TAMARU

Abstract. Ricci soliton contact metric manifolds with certain nullity conditions have recently been studied by Ghosh and Sharma. Whereas the gradient case is well-understood, they provided a list of candidates for the nongradient case. These candidates can be realized as Lie groups, but one only knows the structures of the underlying Lie algebras, which are hard to be analyzed apart from the three-dimensional case. In this paper, we study these Lie groups with dimension greater than three, and prove that the connected, simply-connected, and complete ones can be realized as homogeneous real hypersurfaces in non-compact real two-plane Grassmannians. These realizations enable us to prove, in a Lie-theoretic way, that all of them are actually Ricci soliton.

1. Introduction

A class of contact metric manifolds with certain nullity conditions, so-called the (κ,μ)-spaces, has been introduced by Blair, Koufogiorgos, and Papantoniou ([7]) as follows.

Definition 1.1. Let $(\kappa,\mu)\in\mathbb{R}^2$. A contact metric manifold (M,η,ξ,φ,g) is called a (κ,μ)-*space* if the Riemannian curvature tensor R satisfies

$$R(X,Y)\xi=(\kappa I+\mu h)(\eta(Y)X-\eta(X)Y) \tag{1.1}$$

for any vector fields $X,Y\in\mathfrak{X}(M)$, where I denotes the identity transformation and $h:=(1/2)\mathcal{L}_\xi\varphi$ is the Lie derivative of φ along ξ.

The notion of (κ,μ)-spaces is a generalization of Sasakian manifolds ($\kappa=1$ and $h=0$), and (κ,μ)-spaces have fruitful geometric properties. Among others, (κ,μ)-spaces have strongly pseudo-convex (integrable) CR-structure, and the class of (κ,μ)-spaces is invariant under D-homothetic transformations. We refer to [7] for details. Boeckx ([8]) has also studied (κ,μ)-spaces deeply. He proved

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